WESTMINSTER SCHOOL
THE CHALLENGE 2022

MATHEMATICS II

Tuesday 26 April 2022

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 If $a = 6.3$ and $b = -1.4$, use your calculator to work out $(2a)^2 - b^2 - 2(a-b)^2$.

2 Boris completes his local Park Run in 33 minutes. Of the 5 km course, 3.7 km is tarmac, on which he was able to run at 13 kilometres per hour. The rest of the course is mud. How fast did Boris run through the mud?

3 **a** **i** Two quantities sum to 1. If the first quantity is $\frac{n}{n+1}$, what is the second?

ii By what would you divide $\frac{24w^2}{t}$ to get $\frac{8w}{t^2}$?

b Make T the subject of

$$Q = \frac{H^2}{M - T}.$$

c Simplify

$$\frac{x^2}{2x} - \frac{1}{2}(x-3).$$

d The solution to the equation

$$\frac{x+1}{a} + \frac{3x}{2a} = 5$$

is $x = 7$. Find a.

4 One British Pound Sterling is worth 30 194 Vietnamese Dong.

Before my holiday to Vietnam, I convert £30 into Vietnamese Dong. I am given the exact equivalent as a mixture of notes and coins.

In Vietnam, I only spend my Dong on sixteen bowls of Pho from my favourite street stall. Each bowl costs 29 500 Dong.

When I return, I have some 10 000 Dong notes and some Dong coins. I convert the notes back to Sterling and receive £13.91. How many Dong must I have left over in coins?

5 Three thieves Arnie, Bert and Clive initially divide their loot in the ratio 5 : 3 : 2. Arnie doesn't want to handle the stolen goods, so he divides his share between Bert and Clive, giving Clive £120 more than Bert. Bert and Clive find that their new shares are in the ratio 29 : 26. What was the total value of the loot?

6 Daniel's French teacher has set a number of long vocabulary tests so far this year (each one marked out of 100), and Daniel's average mark is exactly 78%.

Daniel works out that, if he gets 91% on the next test, then his average mark will increase to 79%. In fact, he doesn't revise at all, and his average mark slips to 76%. What mark did he get in the test?

7 **a** Yertie the tortoise's mass increases by 2% each year. If Yertie's mass is 113 kg today, what was it fifty years ago?

b In a certain list of nine numbers, each number is produced by multiplying the previous number in the list by a fixed quantity p.

The first number in the list is $\frac{243}{2}$, and the last number is $\frac{128}{27}$.

Find the middle number in the list.

8 Two thousand steel ball bearings of radius 4 mm are melted down and recast into a solid cylindrical rod. Nine-tenths of the surface area of the rod is curved. Find the length of the rod.

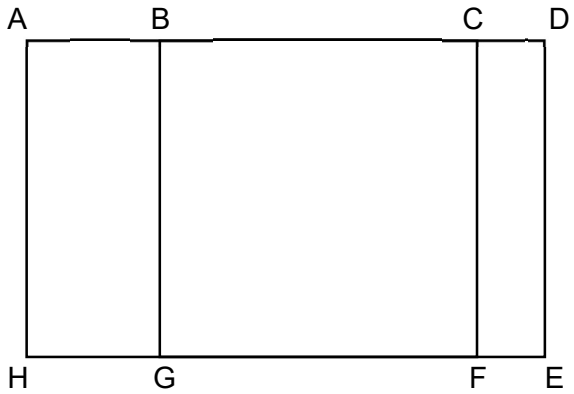
[Reminder: the volume of a sphere of radius r mm is $\frac{4}{3}\pi r^3$ mm^3.]

9 The price of the cryptocurrency Ethereum fell by 51% during December 2021 and rose by 66% during February 2022. Overall, from the start of December 2021 to the end of February 2022, Ethereum prices fell by 32%. What was the percentage change during January 2022?

10 A group of three adults and five children spends £160.05 per week on chocolate. All the adults spend equal amounts, and all the children spend equal amounts.

Financial pressures require the chocolate budget to be changed to £100 per week. Each adult reduces their chocolate consumption by a half; each child, by a third. Find how much each adult and each child spends on chocolate per week after the change.

11 In the diagram, ABCDEFGH is a rectangle and BCFG is a square.
Length AC is 15 cm and length GE is 12 cm.



a Find the perimeter of the rectangle ABCDEFGH.

b If the square occupies two-thirds of the area of the rectangle ABCDEFGH, find the area of the rectangle ABGH.

QUESTIONS CONTINUE OVERLEAF

12 A *Prodonacci* list of numbers is formed as follows:

- the first two numbers in the list are given
- each subsequent number is one more than the product of the previous two

For example, a Prodonacci list might start

5 3 16 49 etc.

because $5 \times 3 + 1 = 16$ and $3 \times 16 + 1 = 49$.

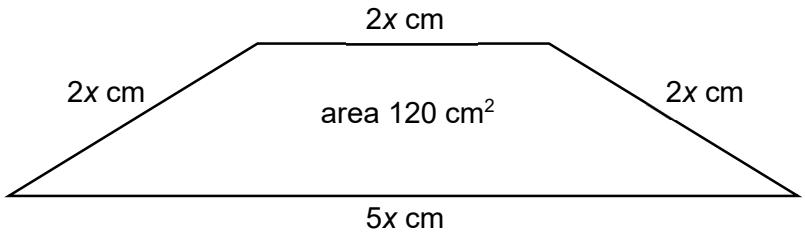
The first number in a different Prodonacci list is 2 and the second is 4.

a Find the next four numbers in the list.

b Is the 2022nd number in the list even or odd? Justify your answer.

c Prove that no number in the list is a multiple of 10.

13 The diagram shows a trapezium with area 120 cm^2.
Find x. Give your answer correct to 3 significant figures.



14 Two hikers, Patricia and Quentin, ascend a hill.
Patricia sets off immediately, walking at a steady speed of 2.5 km per hour.
Quentin also sets off immediately. He walks at a speed of 4 km per hour, but he has to stop for a rest for 5 minutes after every 500 metres.

a If the hill is 2.9 km long, which hiker gets to the top of the hill first, and how much longer does the other hiker take?

b If, instead, the hill is more than 6.5 km but no more than 7 km long, explain why:

i Patricia will definitely beat Quentin;

ii Patricia cannot beat Quentin by more than 6.5 minutes.

c If Patricia beats Quentin to the top of the hill by nine minutes and twelve seconds, what is the shortest possible length of the hill?

END OF QUESTIONS