

WESTMINSTER SCHOOL THE CHALLENGE 2018

MATHEMATICS II

Tuesday 1st May 2018

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper. All your working should be clearly shown. You should attempt all the questions. Please write in black or blue ink.

- 1 Alex had £51.35 when he set out to the shops. He paid for seven muffins at £1.19 each and eleven giant cookies. When he got home, he had £27.84 left. How much was each cookie?
- 2 Sam ran 800 metres in 2 minutes and 28 seconds. Tom ran 1500 metres in 4 minutes and 37 seconds. Which of them ran faster on average? Show the working you do to decide.
- **3 a i** What is the result of adding $\frac{1}{2}(x+1)$ and $x+\frac{1}{2}$?
 - **ii** By what would you need to multiply $\frac{10b}{a}$ to make $\frac{5a}{b}$?
 - **b** Make V the subject of

$$P = \frac{RT}{V-b}.$$

Solve the equation
$$\frac{x+10}{3} - 2(x-1) = 3.$$

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4 The diagram shows a cylinder with a height of 21 cm and a volume of 1336 cm³. What is the surface area of the cylinder?



- **5 a** The volume of a meteorological balloon increases by 14% for each 1000 metres it rises through the atmosphere. The volume of the balloon is 5488 cm³ at a height of 3000 metres. What was its volume at ground level?
 - **b** The pressure in a meteorological balloon decreases by the same percentage, *P*, for each 1000 metres it rises through the atmosphere. The pressure in the balloon is 101300 Pascals at ground level and 34800 Pascals at a height of 8000 metres.

What is the pressure in the balloon at a height of 5000 metres?

[The Pascal is a unit of pressure equal to one Newton per square metre].

- 6 There are 2018 students in a school. Every student must study either History or Geography, but students can study both subjects. The head teacher knows that between 80% and 85% of the students study History, and that between 30% and 40% study Geography.
 - **a** What are the maximum and minimum numbers of students who study Geography?
 - b What are the maximum and minimum numbers of students who study both subjects?

7 The tuck shop sells sandwiches, which all cost the same amount.

David buys eight sandwiches, and pays between £13 and £14 for them.

a If a sandwich costs *K* pence, what is the range of possible values of *K*?

Ezra buys eleven sandwiches, and pays between £17 and £18 for them.

- **b** Find the cost of a sandwich.
- 8 The diagram shows how nine squares have been fitted together to form a rectangle. The smallest square (black) has side length 2 cm and the second smallest square (shaded) has side length *x* cm.
 - **a** Find the side lengths of the other squares, labelled A to G, in terms of *x*.
 - **b** Find the value of *x*, and hence the length and width of the rectangle.



- **9** A Subonacci list of numbers is formed as follows:
 - the first two numbers in the list are given
 - each subsequent number is one less than the sum of the previous two. For example, a Subonacci list could start as follows
 - 2 6 7 12

because 7 = 2 + 6 - 1 and 12 = 6 + 7 - 1.

The first number in a Subonacci list is 4 and the second is 7.

- **a** Find the next four numbers in the list.
- **b** Is the 2018th number in the list even or odd? Justify your answer.
- **c** Prove that none of the numbers in the list is a multiple of three.

10 The diagram shows two straight roads, one heading due North and one heading due East from point O.



Car A heads North and Car B East; they leave O simultaneously.

Car A travels at 27.4 metres per second, and Car B travels at 30.8 metres per second.

a How long does it take before Car A and Car B are 1000 metres apart?

When they are 1000 metres apart, Car A and Car B turn and head directly towards each other, at the same speeds as before.

- **b** How long is it before they collide?
- **11 a** The diagram shows triangle PQR. Angle PQR is a right angle



- PR = 5.3 cm and PQ = 4.5 cm.
- i Find QR.
- ii Find the area of triangle PQR.
- iii Find height *h*, correct to three significant figures.
- **b** The diagram shows a parallelogram ABCD. Angle BAX is a right angle.



AD = 8.7 cm and CD = 4.2 cm. The area of triangle ABX is 8.4 cm². Find the area of trapezium AXCD.

- **12** The point A has co-ordinates (7, 2). The point O has co-ordinates (0,0)
 - B is a reflection of A in the line y = x.
 - C is a reflection of A in the *y* axis.
 - D is a rotation of A by 90° clockwise about O.
 - **a** Work out the co-ordinates of points B, C and D.
 - **b** Find the area of quadrilateral ABCD.

Use the grid below to help you if you wish, but do all your working in the answer booklet.



13 The diagram shows trapezium A and trapezium B.



The height of trapezium A is twice that of trapezium B. The area of trapezium A is three times that of trapezium B. Find *x*.

14 a Find the area of the equilateral triangle shown below.



The circle below has radius R cm. Point O is the centre of the circle and the midpoint of one of the sides of a square of side length R cm.



- **b** Find the total shaded area if R = 27.4.
- **c** Find *R* if the total shaded area is 1445 cm^2 .