HARROW
SCHOOL

Scholarship Examination 2022

Mathematics II

Time: **90 Minutes**

Instructions and advice:

Write your solutions on lined paper, using blue or black ink or pencil. Calculators, geometric instruments (protractor, set square, compass etc.) and squared paper may NOT be used.

Write on only one side of the paper and start your answer to each question on a fresh sheet. Make sure the question number and your name are clearly written on each sheet.

This paper is designed to be very challenging.
Very few (if any) candidates should expect to finish it.
Greater credit will be given for a smaller number of complete solutions to some of the questions rather than a larger number of incomplete attempts.

You do not need to attempt the questions in the order in which they are presented (indeed, you are advised to first read all the questions then start by attempting those with which you feel the most comfortable).

You must show all your working and explain all your reasoning.

PLEASE NOTE: This paper is not just about getting the right answers; correct answers on their own will earn few marks. You will be marked more on the PRESENTATION of your solutions, the EXPLANATION of your working and the JUSTIFICATION of your final answers.

1.

a. Joseph plays tiddlywinks. His mum calculates his mean score across a season to be 35.2 points per match. He asks her "did you remember that my match against Aaron got cancelled at the last minute because he broke his arm?". His mum says that she had forgotten so she recalculates his mean score and gets 36.8 points per match.

How many matches did Joseph play in the season?

b. Henry plays in five cricket matches. The mean number of runs he scores across these five matches is double the mean number of runs he scored in the first three matches. Given that, in total, he scored 160 runs, find the maximum possible number of runs he could have scored in any one of his matches.

2. To raise money for charity, Joss is going to walk a distance equivalent to circumnavigating the globe. Estimate how long this will take him. You may assume that the radius of the earth is 6000km, and should state any further assumptions you make.

3. The number $n!$ (said "n factorial") is equal to $1 \times 2 \times 3 \times \ldots \times (n-1) \times n$.
For example, $4! = 1 \times 2 \times 3 \times 4 = 24$.

a. Work out the value of:

i. $5!$

ii. $(2 \times 3)! - (2! \times 3!)$

iii. $41! \div 39!$ (don't try to work out 41! or 39!)

b. Explain why $\frac{(n+1)!}{(n-1)!} = n^2 + n$, regardless of the value of n.

c. Determine which is larger out of $(n-1)! \times (n+1)!$ and $(n!)^2$.

d. Find a pair of numbers m and n for which $\frac{m!}{n!} = 2022$.

4.

a. Solve $7(2(x+3) - (2-x)) - 4(2x-5) = 4(x+16)$

b. Hence solve:

i. $7(2(a^2+3) - (2-a^2)) - 4(2a^2-5) = 4(a^2+16)$

ii. $7(2(\sqrt{b}+3) - (2-\sqrt{b})) - 4(2\sqrt{b}-5) = 4(\sqrt{b}+16)$

iii. $7\left(2\left(\frac{2}{c}+3\right) - \left(2-\left(\frac{2}{c}\right)\right)\right) - 4\left(\frac{4}{c}-5\right) = 4\left(\frac{2}{c}+16\right)$

5. This question involves *factorial numbers*, which are defined in Q3.
We can think about the number of 0s on the end of $n!$. For example, $11! = 39{,}916{,}800$, which has **two** 0s on the end.

a. The last nine digits of 34! are … 520,000,000. How many 0s do:

i. 33!

ii. 35!

have on the end?

b. Determine, with justification, how many 0s there are on the end of 20!

c. How many positive whole numbers n there for which $n!$ doesn't contain any 0s? Justify your answer.

6. Tim likes to go on boat journeys. His boat always moves at the same speed relative to the water. Last weekend he went boating in a river which was flowing at a constant rate. He first travelled upstream (i.e. *against* the flow of water) for 3½ hours, before turning round and travelling downstream (i.e. *with* the flow of water) back to where he started. The return leg of his journey took him 2½ hours.

a. Find the ratio of the speed of the boat (in still water) to the speed at which the river was flowing.

b. Find how long it would take the boat to travel the same distance in still water.

7. You are given that $145 \times 290 = 42{,}050$.
<u>Using this fact</u>, work out the following:

a. 14.5×29

b. $42{,}050 \div 0.29$

c. 155×290

d. 145^2

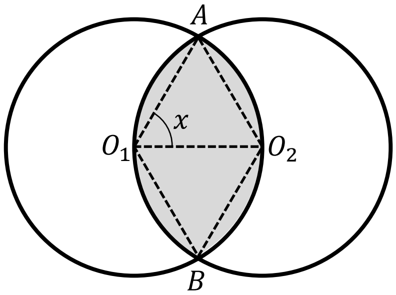
e. $\sqrt{420.5 \div 50}$

f. Find a pair of numbers A and B for which $\frac{435}{42050} = \frac{1}{A} + \frac{1}{B}$.

NB You will earn very few marks for working these out directly – this question is about how you use the fact you are given.

8. In the diagram below, the two circles each have radius 2cm and pass through each other's centre (labelled O_1 and O_2 respectively). The two points where the circles intersect are labelled A and B.

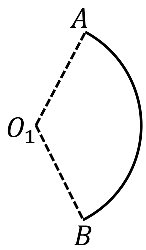
Definition: A ***sector*** is a part of a circle created by two straight cuts from its edge to its centre (e.g. ,  and )



a. What is the size of the angle labelled x? Justify your answer.

b. By first working out its height, find the area of the triangle AO_1O_2. You should leave a square root in your answer.

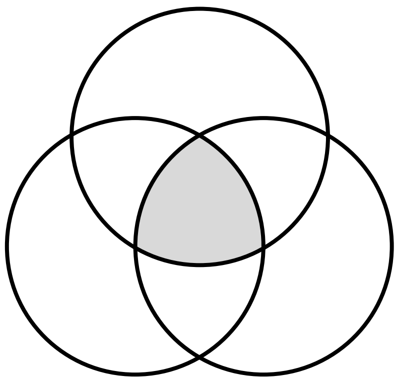
c. By thinking of it as a fraction of the circle with centre O_1, work out the area of the sector O_1AB (shown in the diagram below). You should use $\pi = \frac{22}{7}$.



d. By considering your answers to parts (b) and (c) together, find an expression (involving a square root) for the area of overlap between the two circles (shaded in the diagram above).

In the diagram below, the three circles each have radius 2cm and the centre of each circle lies is at a point of intersection of the other two circles.

e. Find an exact expression for the area of overlap between the three circles (shaded in the diagram).



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