



HARROW SCHOOL

Scholarship Examination 2021

Mathematics II

Time: **90 Minutes**

Instructions and advice:

Write your solutions on lined paper, using blue or black ink or pencil. Calculators, geometric instruments (protractor, set square, compass etc.) and squared paper may NOT be used.

Write on only one side of the paper and start your answer to each question on a fresh sheet. Make sure the question number and your name are clearly written on each sheet.

This paper is designed to be very challenging.

Very few (if any) candidates should expect to finish it.

Greater credit will be given for a smaller number of complete solutions to some of the questions rather than a larger number of incomplete attempts.

You do not need to attempt the questions in the order in which they are presented (indeed, you are advised to first read all the questions then start by attempting those with which you feel the most comfortable).

You must show all your working and explain all your reasoning.

PLEASE NOTE: This paper is not just about getting the right answers; correct answers on their own will earn few marks. You will be marked more on the PRESENTATION of your solutions, the EXPLANATION of your working and the JUSTIFICATION of your final answers.

1. Calculate the following, showing any intermediate steps or methods you use:

- a. 375×48
- b. $477777773 \div 43$
- c. $\frac{1}{0.01^3}$
- d. $3\frac{13}{33} \div 12\frac{8}{11}$
- e. 35% of 85% of $\frac{5000}{119}$
- f. $\sqrt{3\frac{1}{16}}$
- g. $\frac{1}{\frac{1}{7} \frac{1}{9}}$

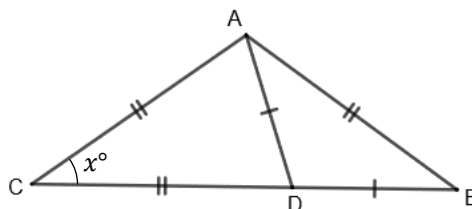
2. There is a story about the German mathematician Carl Gauss in which, when he was eight years old, his school teacher asked the class to work out $1 + 2 + \dots + 99 + 100$. He stunned his teacher by writing down the right answer only a few seconds after the teacher had finished asking the question. He achieved this feat by recognising that the question is greatly simplified by pairing up the numbers in a particular way – the first with the last, the second with the last-but-one and so on.

- a. When paired up in this way, what is the sum of each pair?
- b. How many such pairs are there?
- c. Using your answers to parts (a) and (b), deduce Gauss' answer.
- d. Find the following sums:
 - i. $1 + 2 + \dots + 9999 + 10\,000$
 - ii. $1 + 2 + 3 + \dots + (n - 1) + n$ (where n is an integer)
 - iii. $2 + 6 + 10 + 14 + \dots + 250$
 - iv. $101 + 102 + 103 + \dots + 300$
 - v. $1 + 2 + 4 + 5 + 7 + 8 + \dots + 995 + 997 + 998 + 1000$ (this sum is missing all the multiples of 3)

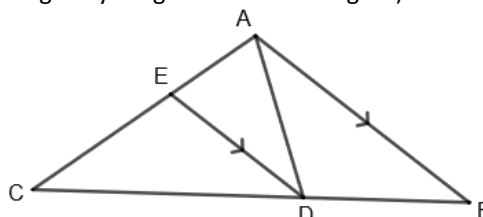
3. Estimate, to one significant figure, the number of times your heart will beat in your lifetime. You will need to state any assumptions you make.

4. Jonah has drawn a geometric diagram.

- a. In his diagram (below), CDB is a straight line and the marked sides are equal. What is the size of the angle x ? Ensure you explain each step of your reasoning.



- b. Jonah then draws an extra line in his diagram which passes through D and is parallel to AB , as shown in the diagram below (he doesn't change anything else about the figure).



Determine, justifying your answer, the value of the ratio
 Area of ABD : Area of CDE

5. Maths-itis is a very rare disease, which affects one person in every 10 000. Professor Stewart has come up with a test for Maths-itis which is 99% effective (i.e. if it says you have Maths-itis it is right 99% of the time and if it says you don't it is right 99% of the time).

You use Professor Stewart's test to test a million people for Maths-itis.

- How many of the million people would you expect to have Maths-itis?
- Of these people, how many of them will be told they have it when they are tested?
- Of the people who *don't* have Maths-itis, how many will be told they *do* have it when they are tested?
- Of the people who were told they did have Maths-itis when they were tested, how many actually have it?
- Would you say Professor Stewart's test is a good one?

6. A number is divisible by 9 if and only if the sum of its digits is divisible by 9. For example:

94 815 is divisible by 9, since its digit sum is 27 ($= 3 \times 9$), but
1575 is not, since its digit sum is 17 which is not divisible by 9.

A number is divisible by 11 if and only if the difference between the sums of its alternate digits is divisible by 11. For example:

$\underline{1} \overline{7} \underline{2} \overline{9} \underline{3} \overline{2} \overline{0} \underline{7} \underline{2} \overline{5}$ is divisible by 11, because $\underline{1} + \underline{2} + \underline{3} + \underline{0} + \underline{2} = 8$, $\overline{7} + \overline{9} + \overline{2} + \overline{7} + \overline{5} = 30$, and the difference between 8 and 30 is 22, which is a multiple of 11;

406 802 is divisible by 11, since the difference between $(4 + 6 + 0)$ and $(0 + 8 + 2)$ is 0, which is a multiple of 11; but
836 910 is not, because the difference between $(8 + 6 + 1)$ and $(3 + 9 + 0)$ is 3, which is not a multiple of 11.

- Show that:
 - 9 876 543 210 is divisible by 9
 - 40 506 070 is divisible by 11 but not by 9
 - 12 345 654 321 is divisible by 99.
- The number $2d7\ 852$ (where d is an unknown digit) is divisible by 11. Find the missing digit d .
- The number $48\ 12e\ 453$ is divisible by 9. Explain why there are two possible values for the missing digit e and state what they are.
- The number $17a\ 35b$ is divisible by both 9 and 11. Find the missing digits a and b .

7. You are running through the train station to catch a train when you notice your bag is broken. In order to fix it, you will need to stop running for ten seconds. Ahead of you is an escalator (which takes more than ten seconds to travel up).

You have two options:

- You could stop running now, fix your bag then run to the train (running up the escalator), or
- You could run to the escalator, then once on it fix your bag before running the rest of the way to the train (including the running up the remainder of the escalator).

Which of these would you choose to minimise the time it takes you to get to your train? Justify your answer.

(You may assume that you run at a constant speed and that the escalator travels at a constant speed.)

8. In this question, n represents a positive integer. Find all possible values of n for which the following are true:

- $\frac{12}{n}$ is an integer
- $\frac{20}{n+1}$ is an integer
- $\frac{30}{2n-3}$ is an integer
- $\frac{2n}{n-6}$ is an integer (Hint for (d): $2n = 2n - 12 + 12$)