Eton College King's Scholarship Examination 2019

MATHEMATICS A

(**One and a half hours**)

Candidate Number:………………………………………………

Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:
> *Section I (Short-answer questions) – 50 marks available*
> *Section II (Extended questions) – 50 marks available*

Answer all of Section I and as many questions as you can from Section II.

The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

Neither calculators nor protractors may be used.

ADDITIONAL MATERIALS: NONE

Do not turn over until told to do so.

Section I : Short-answer questions (50 marks)

1. Find the value of the following, giving your answers as reduced, mixed fractions:
   a) $1\frac{2}{13} \times 1\frac{1}{25}$

   [3]

   b) $1\frac{5}{12} \div 3\frac{13}{24}$

   [3]

   c) $345\frac{2}{3} - 327\frac{7}{9}$

   [3]

2. Find the value of the following, giving your answer as a decimal or whole number as appropriate:
   a) 3400×0.003

   [3]

   b) $0.00396 \div 0.000003$

   [3]

   c) 0.01^4

   [3]

3. The bearing of Blackburn from Burnley is 257°. What is the bearing of Burnley from Blackburn?

[2]

4. 6 children have a mean pocket money of £5. The four boys have a mean pocket money of £7; what is the girls' mean pocket money?

[3]

5. Solve the following equation and inequality, giving your final answers as a reduced, mixed fraction (where appropriate) and with x on the left-hand side:
    a) $3(x - 7) - 4(x - 8) = 7(x - 11)$

[3]

    b) $11(x + 3) \geq 19x - 9$

[3]

[Page 3 of 15]

6.   Find the length of the line segment joining the points $(-3, 13)$ and $(2, 1)$.

[3]

7.   An alien fires a rocket upwards from the surface of his home planet; t seconds after it has left the ground, the height, h metres, of the rocket above the ground is given by the formula:

$$h = 2000t - 25t^2$$

Find, in metres, the distance the rocket travels
a)   over the first four seconds;

[2]

b)   over the next four seconds.

[3]

[Page 4 of 15]

8.   Evaluate the following, giving your answer in standard form:
     a) $7.41 \times 10^{21} - 6.82 \times 10^{21}$

[2]

     b) $9.2 \times 10^{-6} + 9.1 \times 10^{-7}$

[3]

9.   Hubert is packing rulers into boxes. If he packs them into boxes of 12 each, all the boxes
     are full and no rulers are left over; similarly, he can pack them into boxes of 16 each,
     filling them all with none left over. However, if he packs them in boxes of 5, he has one
     ruler left over. Find the smallest number of rulers that Hubert could have.

[4]

10.   Lara monetizes her social media accounts with both Facetube and Instachat. She posts a video on Facetube and earns a fixed amount for each 'like' she receives; also, she receives a fixed amount for every new friend acquired on Instachat. In the first week, she gets 5 likes for her video and gains 6 new friends on Instachat: she receives £1.13. The next week she gets 6 likes for her video, gains 7 new friends on Instachat and receives £1.33. If, in the third week, she gets 8 likes for her video and gains 8 new friends, how much money will she receive?
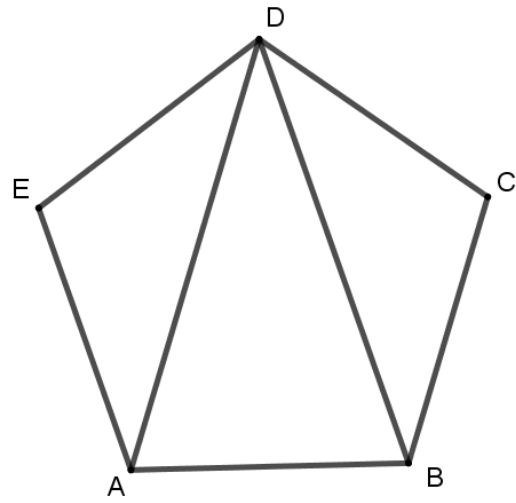
[4]

Section II: Extended questions (50 marks)

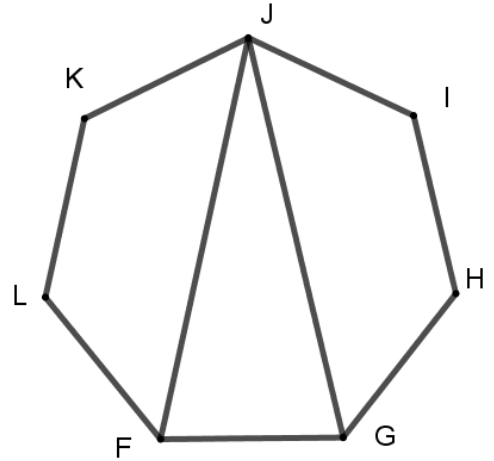11. *No credit will be given in this question if angles are measured or estimated.*

   a)   The diagram on the right shows a regular pentagon *ABCDE* with two diagonals *AD* and *BD* drawn.

   Show clearly that the ratio of angles $\angle DAB : \angle ADB$ is $2:1$.

[3]

b) The second diagram shows a regular heptagon *FGHIJKL* with two diagonals *FJ* and *GJ* drawn. Find the ratio of angles $\angle JFG : \angle FJG$.

[7]

12.   In this question, the letters *a* and *b* represent digits in the normal decimal representation of a number. For example, '52*ab*' could represent any whole number from 5200 to 5299 inclusive.

   a)   The four-digit number '537*a*' is divisible by 4. Write down both possible four-digit numbers.

[2]

   b)   The five-digit number '7218*a*' is divisible by 9. Write down both possible five-digit numbers.

[2]

   c)   The four-digit number '*a*37*b*' is divisible by 18. Find all possible four-digit numbers.

[3]

   d)   The seven-digit number '1*a*234*b*2' is divisible by 36. Find all possible seven-digit numbers.

[3]

13. On the right is an example of a *magic square*: each row, column and diagonal sum to the same value, called the *magic number*.

For example:
$8 + 7 + 6 = 21$ (row)
$9 + 8 + 4 = 21$ (column)
$9 + 7 + 5 = 21$ (diagonal)

9	2	10
8	7	6
4	12	5

The magic number, in this case, is 21.

a) Fill in the remaining gaps in the magic square below and state the magic number.

15	0	18
	22	7

[2]

b) Below is an algebraic magic "square"; find the magic number (which will be an algebraic expression) and hence fill in the gaps:

$x - y$	$x + y - z$	$x + z$
$x + y + z$	x	
	$x - y + z$	

[3]

c) Using the algebraic magic square or otherwise, fill in the remaining gaps in the magic square below:

		25
	18	16

[5]

14. Tom, Dick and Harry all have rather inaccurate digital watches, with times displayed in the 24-hour clock with seconds.

- Tom's watch loses five minutes every hour.
- Dick's watch gains an hour every day.
- Harry's watch gains 3 seconds every minute.

All three boys set their watches correctly at 6 o'clock on Monday morning.

a)   When the correct time is exactly 7am, Tom's watch shows "06:55:00". What do Dick and Harry's watches show?

[2]

b)   At 6am on Tuesday, Dick's watch shows "07:00:00". What do Tom and Harry's watches show?

[2]

c)   Later on Tuesday, Dick's watch claims it is four hours later in the day than Tom's watch. What time does Tom's watch display?

[2]

[Page 12 of 15]

d) The following week, Tom and Harry's watches appear to be displaying the same time. On what day does this happen, and what time is displayed?

[4]

15. a) A triplet of positive whole numbers (a,b,c) is called a *Pythagorean triple* if

$$a^2 + b^2 = c^2$$

Complete the rows of examples of Pythagorean triples in the following table:

a	b	c
3	4	5
	12	13
8		17
	24	25
18		30
15	36	

[3]

b) The diagram shows a kite *MNOP* of perimeter 128cm; $ON = OP = 25$ cm; the point *Q* lies on *OP* such that $\angle NQP$ is a right angle. The area of triangle *OPN* is 300cm^2.

i.   Find the length *NQ*.

ii. Find the length *NP*.

iii. Find the area of the kite.

[7]

END OF PAPER

[Page 15 of 15]