# Eton College King's Scholarship Examination 2018 

## MATHEMATICS A

(One and a half hours)

## Candidate Number:

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## Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:
Section I (Short-answer questions) - 50 marks available
Section II (Extended questions) - 50 marks available
Answer all of Section I and as many questions as you can from Section II.
The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

The use of calculators is NOT permittted.

## Do not turn over until told to do so.

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## Section I : Short-answer questions (50 marks)

1. Find the value of the following, giving your answers as reduced, mixed fractions:
a) $1 \frac{13}{14} \times 5 \frac{4}{9}$
b) $3 \frac{21}{35} \div 1 \frac{13}{35}$
c) $\quad 207 \frac{3}{10}-199 \frac{7}{15}$
d) $\left(3+\frac{1}{2}\right)^{2}$
$\qquad$
2. Find the value of the following, giving your answer as a decimal or whole number where appropriate:
a) $3.5 \times 0.04$
b) $0.064 \div 0.000016$
c) $\quad(0.04)^{3}$
3. Solve the following equation, giving your answer as a mixed fraction:

$$
\begin{equation*}
13 x-7=1+5(3-x) \tag{3}
\end{equation*}
$$

$\qquad$
4. Solve the following inequality. In your final answer, $x$ must appear on the lefthand side.

$$
\begin{equation*}
19 x-3<23 x+21 \tag{3}
\end{equation*}
$$

5. Solve the pair of equations simultaneously:

$$
\begin{align*}
& 3 x-4 y=27 \\
& 4 x-3 y=43 \tag{4}
\end{align*}
$$

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6. A square has an area of $0.0036 \mathrm{~cm}^{2}$. Find its perimeter, in centimetres.
7. I have a block of gold weighing five-twelfths of an ounce. I remove fourfifteenths of an ounce. What percentage of the original gold remains?
8. Most of the children in a certain school were born in one of five English counties. A third were born in Berkshire, a fifth were born in Buckinghamshire, a sixth were born in Hampshire, an eighth were born in Surrey and a tenth were born in Middlesex. The remaining 45 children were born in other counties. How many children are in the school?
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9. A square is divided into four rectangles as shown; the areas of three of the rectangles are given. Find:
i) the area of the remaining rectangle;

ii) the side length of the square.
10. In a survey, people were asked if they owned a goat or a camel. One person in fifteen said they had a goat; one person in eighteen said they had a camel and a tenth of the people had one animal or the other but not both. What proportion of the people owned neither kind of animal?
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## Section II: Extended questions (50 marks)

11. Note: The notation $5 \sqrt{7}$ is merely shorthand for $5 \times \sqrt{7}$
a) Show that $5 \sqrt{7}$ equals $\sqrt{175}$.
b) Without trying to work out the square roots as decimals, try to determine which of the two is the larger: $3 \sqrt{3}$ or $2 \sqrt{7}$.
c) An equilateral triangle $A B C$ has sides of length 12 units. Use Pythagoras' Theorem to show that the perpendicular height $C D$ is $a \sqrt{3}$ units, where $a$ is a whole number to be found.

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d) A square-based pyramid ABCDV is made from a square of side 12 cm and four equilateral triangles as shown. Find the perpendicular height $V M$, where $M$ is the centre of $A B C D$, and show it can be written as $b \sqrt{2}$ where $b$ is a whole number to be found.

12. a) $A B C D$ is a trapezium. The internal angle at $A$ is $9^{\circ}$ greater than the internal angle at $B$ but half the internal angle at $C$. Find the internal angle at $D$.

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b) In the diagram shown, the line segments $A B, B C, C D$ and $D A$ are all equal in length; in addition, $B E=B D$. Lines $A E D, B E F$ and $C D F$ are straight. Given that $\angle B F D=33^{\circ}$, find the value of $\angle F E D$.

13. a) The letters $a, b, c, d$ and $e$ represent five different positive, whole numbers such that $a b c d e=210$. Show (clearly) that there is exactly one possible value for $a+b+c+d+e$ and find that value.
b) Let $p, q, r$ and $s$ represent four different positive, whole numbers such that pqrs $=210$. Find all possible values for $p+q+r+s$.
c) Let $w, x, y$ and $z$ represent four whole numbers (not necessarily positive).

If $(w-1)(x-2)(y-3)(z-4)=10$, find all possible values for $w+x+y+z$
[5]
14. a) If $x$ and $y$ are positive, whole numbers, then two possible solutions to the equation $50 x+9 y=2018$ are $(x, y)=(4,202)$ and $(x, y)=(13,152)$.
i) Verify these are indeed solutions to the equation.
ii) Find the other three solutions to the equation.
b) I buy chocolate bars and fizzy drinks. Of the chocolate bars, Venus bars cost 68 p each and Tiger bars cost 48p each; cans of fizzy drink cost 60p each. I buy five times as many chocolate bars (Venus and Tiger combined) as I buy cans of fizzy drink. If I spent $£ 10.40$, how many cans of fizzy drink did I buy?
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15. a) $A B$ is a diameter as shown. $O$ is the centre of the circle. Prove that $\angle A C B$ is a right angle.

b) In triangle $P Q R$ represented below, $R Q=40 \mathrm{~cm}, R P=30 \mathrm{~cm}$ and both $\angle P R Q$ and $\angle P S R$ are right angles. Find the perpendicular height $R S$.

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c) In the diagram shown, triangle $J K L$ is isosceles with $K J=L J=10 \mathrm{~cm}$ and $K L=12 \mathrm{~cm}$. The small circle touches $K L$ at its midpoint $H$ and touches the larger circle at G in such a way that the line $G H J$ is a line of symmetry. Find the radius of the smaller circle.


