Eton College King's Scholarship Examination 2016

MATHEMATICS A

(One and a half hours)

Candidate Number:.....

Please answer on the paper in the spaces provided.

Please write your candidate number on every sheet.

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all your working.

The use of calculators is NOT permittted.

<u>ADDITIONAL MATERIALS</u>: ***None***

Do not turn over until told to do so.

1. This question is compulsory.

   a) Solve the following equations:

      i) $27x - 12 = 11x + 20$ [2]

      ii) $\frac{x}{2} + \frac{3}{4} = \frac{x}{5}$ [2]

   b) Calculate the following, leaving your answers as mixed numbers in their lowest terms:

      i) $\frac{72}{65} \times 1\frac{1}{12}$ [2]

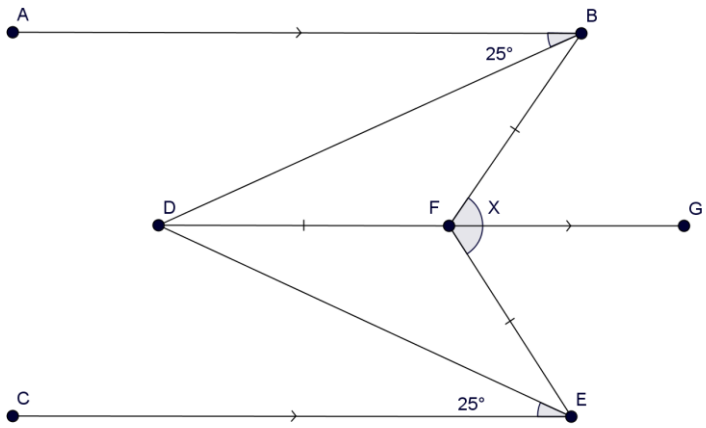
      ii) $2016\frac{2}{3} - 2014\frac{3}{4}$ [3]

   c) This question is about mean scores.

      i) Show that $13 \times 9.9 = 128.7$ [2]

ii) Two classes do the same test. The mean score of the 12 children in the first class is 12.4 and the mean score of the 13 children in the second class is 9.9. What is the mean score of the 25 children? [4]

d) The diagram shows 3 parallel straight lines AB, DFG and CE. The lengths BF, DF and EF are the same, and both angles ABD and CED are 25^o. Find, giving reasons, the shaded angle BFE, labelled X.



[3]

e) Solve the simultaneous equations:

$$4y = x - 14$$
$$2y = 5x - 16$$

[4]

f) I am given a rectangle A.

i) Rectangle B is created by increasing the length of rectangle A by 12% and decreasing the width of the rectangle by 10%. Show that the area of rectangle B is 0.8% larger than the area of A. [2]

ii) Rectangle C is created by increasing the length of rectangle A by 25% and decreasing the width of the rectangle. If the two rectangles A and C have the same area, by what percentage was the width of rectangle A decreased? [3]

g) You are given the formula $x = \frac{a}{2b^2} + \frac{2b}{a^2}$

i) Calculate the value of x when $a = 1\frac{1}{2}$ and $b = 3$, giving your answer as an exact fraction. [3]

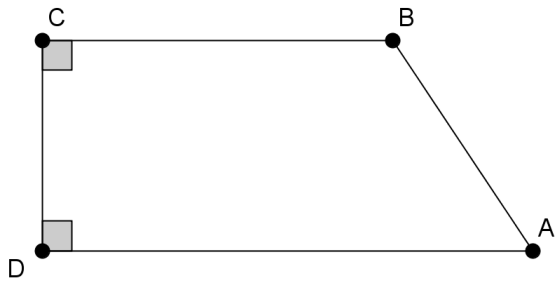
ii) Suppose that $b = a^2$ and $x = 6$. Find the value of a. [3]

h) What do I need to subtract from $p-2q+3r$ to get

i) $p+2q-3r$ [2]

ii) $3p-2q+3r$ [2]

i) The diagram shows a trapezium ABCD. The lengths AD, DC and BA are 12cm, 6cm and 6.5cm respectively. Calculate the area of ABCD. [4]



j) Solve the following inequalities:

i) $-7x-4<4-3x$ [2]

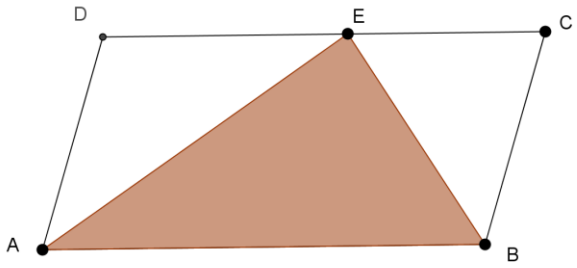
ii) $\frac{2}{3}(4-3x)-\frac{8-6x}{5}>1$ [3]

k) Rachel thinks of a whole number between 1 and 2016. She divides it by 5, adds 4 and then divides by 0.6 to get an answer which is a whole number. How many numbers could she have started with? [4]

[Total for Question: 50]

2. This question is about the areas of shapes.

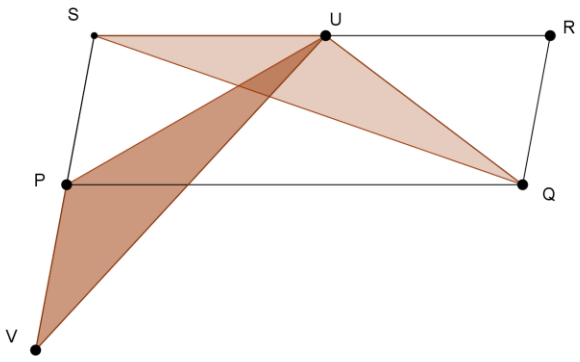
a) The diagram shows a parallelogram ABCD of area $180cm^2$. The point E lies on the side CD.



i) Write down the area of the triangle ABE. [1]

ii) Suppose the lengths DE and EC are in the ratio 11:7. Calculate the area of the triangles AED and BCE. [3]

b) The diagram below shows a parallelogram PQRS. U is the midpoint of RS and P is the midpoint of SV. Show, giving reasons, that the shaded triangles QUS and VUP have the same area. [6]



[Total for Question:10]

3. This question is about repeated division.

   a) Calculate $(4 \div 5) \div 20$ and $4 \div (5 \div 20)$. [1]

   b) a, b and c are positive whole numbers. If $(a \div b) \div c = a \div (b \div c)$, what can be said about the value of c? [2]

   c) There are five ways to put two pairs of brackets into $a \div b \div c \div d$. Two of them are $(a \div b) \div (c \div d)$ and $((a \div b) \div c) \div d$.

      i) Find the other three. [3]

      ii) Show that two of them always give the same answer. [4]

[Total for Question: 10]

4. This question is about means.

   a) Find the mean of the numbers

      i) 1, 2, 3, 4, 5, ……99 [1]

      ii) 2, 3, 4, 5, 6, ……100 [1]

   b) Sam writes down the numbers 1, 2 ,3…….100 and deletes one number *n*. The mean of the remaining numbers is a whole number. Explain carefully why *n* must be either 1 or 100. [2]

   c) Michael writes down the numbers 1, 2, 3……101 and deletes one number *m*. The mean of the remaining numbers is a whole number. Find, giving reasons, the possible value(s) of *m*. [6]

[Total for Question:10]

5. I have two whole positive numbers, x and y.

a) For each of the following, circle the correct answer, giving a brief reason:

i) If $(x + y)$ is odd , then $(x – y)$ is

A Even B Odd C Depends [2]

ii) If $(x + y)$ is even , then $(x – y)$ is

A Even B Odd C Depends [2]

b) Expand $(x – y)(x + y)$ [1]

c) I have a large square pond set inside a square garden: both the pond and the garden have sides which are a whole number of metres, and outside the pond the garden is grassed over. The area covered by grass is 188 square metres. Find the area of the pond.

[5]

[Total for Question:10]

6. In this question, a string number is a number whose digits are either 0 or 1. A string number can be changed into a new one using any one of the following four changes

X: replace 0 by 110	Z: replace 1 by 010
Y: replace 110 by 0	T: replace 010 by 1

a) Show it is possible to change the string number 01111 to the string number 0. You need to indicate your changes clearly. [2]

b) For each of the following, explain why it is impossible to change the string number 0 into the given number:

i) 0101 [3]

ii) 10101 [5]

[Total for Question: 10]

END OF PAPER