Eton College King's Scholarship Examination 2015

MATHEMATICS A

(One and a half hours)

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all your working.

The use of calculators is NOT permittted.

Remember to write your candidate number on every sheet of answer paper used.

Do not turn over until told to do so.

1.   This question is compulsory.

(a)   Given that $a = -3$ and $b = 5$, calculate the following:

     (i) $b^2 - a^2$

[2]

     (ii) $\dfrac{2b^2}{a-2}$ [3]

(b)   Calculate the following, leaving your answers as simplified mixed fractions:

     (i) $10\frac{5}{8} - 3\frac{1}{40}$ [3]

     (ii) $8 \div 1\frac{5}{11}$ [3]

(c)   Simon wishes to sell his car for £3,200.

     (i)   If Simon agrees to give me a 15% discount, how much would I have to pay for the car?   [2]

     (ii)   If I buy the car at this discount and then sell it back to Simon for 15% more than I paid for it, how much do I sell it for?   [2]

(d)   Evie is not very good at using a protractor. When she measures an angle, she always ends up with a value which is 180° minus the correct value (e.g. if the angle is 40°, she measures it as 140°). If she measures the angles of a triangle and adds them up, what answer will she get?   [3]

(e)   Solve the following equations, giving your answers as simplified fractions:

     (i) $x - 2(7 - 3x) = 4$ [3]

     (ii) $\dfrac{2}{3}(y+2) - \dfrac{y}{6} = \dfrac{41}{24}$ [4]

(f)   Solve the simultaneous equations:

$$5x - 2y = 12$$
$$7x + 3y = 11$$

[4]

(g)   (i)   The mean average of 3 numbers is 4.7. If two of the numbers are both 3.1, what is the third number?   [2]

     (ii)   The smallest number in a group of 3 numbers is 2.1 less than the middle number and 4.5 less than the biggest number. How much larger than the smallest number is the mean average of all 3?   [3]

(h)   Solve the following inequalities:

(i) $4 + x < \dfrac{1}{2}x$ [3]

(ii) $y - (1 - y) < 1 + y$ [3]

(i)   Iris has £5,000 to invest. She invests some of it in a company called Safehouse and the rest in a company called Riskybusiness. After one year her investment in Safehouse has increased by 20% but her investment in Riskybusiness has decreased by 20%. If her total investment after one year is now worth £5,300, use algebra to find how much she initially invested in Safehouse.   [4]

(j)   (i)   What do I add to $3 - 2x$ to get $x - 2$?   [2]

        (ii)   By what do I multiply $\dfrac{2}{x}$ to get $\dfrac{x}{2}$?   [2]

        (iii)   By what do I divide $\dfrac{y}{x}$ to get $\dfrac{x}{y}$?   [2]

2.   (a)   A triangle PQR has two sides PQ and QR which are equal in length. Given that the angles at P, Q and R are $(x + y)^{\circ}$, $(x + 2y)^{\circ}$ and $(3x - y)^{\circ}$ respectively, find x and y.   [4]

      (b)   The diagram shows an isosceles triangle ABC in which AB = AC and angle BAC = 20°. The points X and Y are on the sides AB and AC respectively, and BC = CX = CY.
              (i)   Giving reasons, calculate the angles ABC and XCB.   [3]
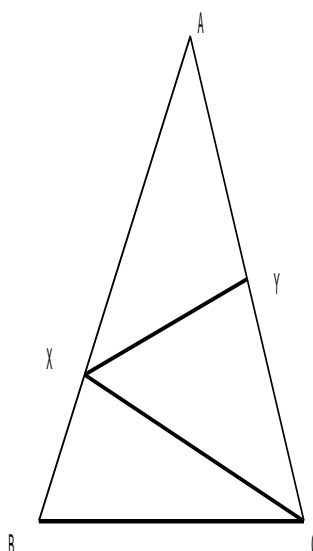   (ii)   Prove that XY = BC.   [3]

Diagram Not to scale

3.   (a)   Show that $(10{,}000 + a)^2 = 100{,}000{,}000 + 20{,}000a + a^2$.   [2]

      (b)   Show there are 4 digits which are 0 in the square of ten thousand and seven.   [2]

      (c)   A quattuordecillion is 10^{45}. How many zeroes are there in the square of one quattuordecillion and seven.   [3]

      (d)   If the number x is two fewer than a quattuordecillion, how many zeroes are there in x^2 ?   [3]

4.   (a)   In the diagram below, A, B and C lie on a straight line, and angle DAB is 90°.
             DC = 52 cm, DB = 25 cm and AC = 48 cm.
             (i)   Find the length DA.   [3]
             (ii)   Find the length AB.   [2]

Diagram Not
to scale
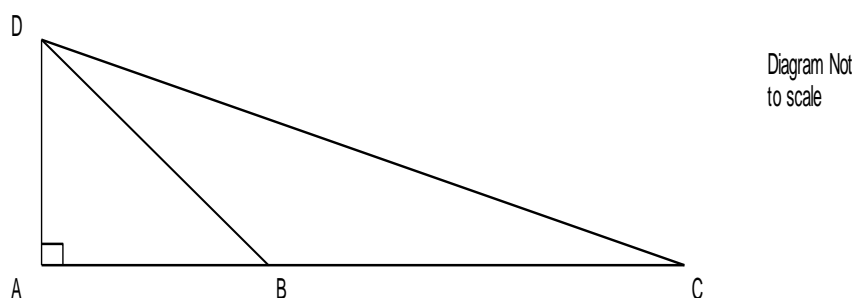
      (b)   In the diagram below, the points P, Q and N lie on a straight line and the points
             R, S and N lie on a second straight line. The angle PNR is 90°.
             Show that $PR^2 + QS^2 = PS^2 + QR^2$.   [5]
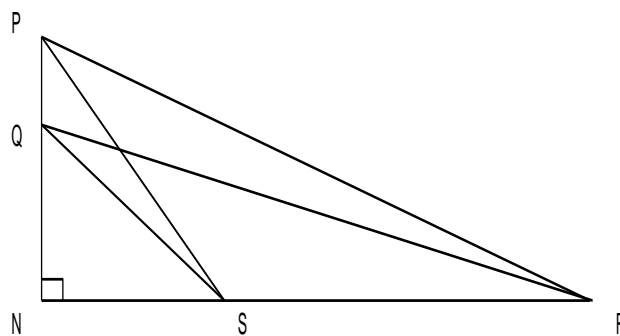
5. A derangement is a re-ordering of the members of a set such that none of the members appear in their original position. For example BCAED is a derangement of ABCDE, but BCEDA is not (as the D remains in its original position).

   (a) Write down both derangements of ABC. [1]

   (b) (i) Write down all the derangements of ABCD which start with a B [2]
       (ii) Show there are 9 derangements of ABCD. [2]

   (c) There are 11 derangements of ABCDE which start with a B. How many derangements are there of ABCDE in total? [2]

   An almost-derangement is a re-ordering of the members of a set such that exactly one of the members appears in its original position. For example BCEDA is an almost-derangement of ABCDE.

   (d) How many almost-derangements of ABCDEF are there? [3]

6. (a) For any two whole numbers n and m, we define $n\nabla m$ as follows:
$$n\nabla 1 = 1;$$
$$n\nabla m = n\nabla(m-1) + n.$$
   For example,
$$5\nabla 1 = 1,$$
$$5\nabla 2 = 5\nabla 1 + 5 = 6;$$
$$5\nabla 3 = 5\nabla 2 + 5 = 11.$$
       (i) Show that $7\nabla 4 = 22$. [2]
       (ii) Find an expression for $n\nabla 6$ in terms of n. [2]

   (b) For any two whole numbers n and m, we define $n\angle m$ as follows:
$$n\angle 1 = n;$$
$$n\angle m = n\angle(m-1) + m.$$
   For example,
$$5\angle 1 = 5,$$
$$5\angle 2 = 5\angle 1 + 2 = 7$$
$$5\angle 3 = 5\angle 2 + 3 = 10.$$
       (i) Show that $5\angle 7 = 32$. [2]
       (ii) For how many different values of n does the sequence
$$n\angle 1, n\angle 2, n\angle 3, n\angle 4\ldots\ldots$$
       contain exactly two numbers between 95 and 100 inclusively. [4]

END OF PAPER