Eton College King's Scholarship Examination 2014

MATHEMATICS B   **(One and a half hours)**

Answer as many questions as you can.

Each of the ten questions carries ten marks.

Show all your working.

Calculators are allowed.

Do not turn over until told to do so.

1. (a) How many seconds are there in a ninth of a week?

    (b) Ann, Jan and Dan all came joint 27th in the year with each other (and with no other in the year) in the maths test but this also put them joint 27th from the bottom. How many are in the year?

    (c) I think of three whole numbers which are all different from each other. They add to make 1 but multiply to make 36.

        (i)   Explain why at least one of the numbers must be negative.

        (ii) Explain why exactly two of the numbers must be negative.

        (iii) Find all three numbers.

2. (a) I have w kilogrammes of wheat. I discard $\frac{2}{3}$ of it and divide the remainder in the ratio of 1:3 between Harry and Hugo. If each pays me p pounds per kilogramme, how much does Hugo pay me?

    (b) I have f kilogrammes of flour. I discard 5% because it is bad. I divide the remainder up in the ratio of 4:7:8 between Jack, Jill and Jake who each pay me w pounds per kilogramme. Jill gives Jake y kilogrammes of her flour; Jake sells all of his flour for z pence per 100 grammes.

        How much profit does Jake make?

3. A coach party of adults and children go on a fairground ride. They fill up 15 cars on the ride so that each car has either exactly 6 children or exactly 5 adults. A ticket for each adult is £3.20 and for a child is £2.25; the total cost for all is £220.

    (a) Lara writes down one equation:

$$\frac{x}{5} + \frac{y}{6} = 15$$

    What does she mean by x and y?

    (b) Write down a second equation involving x and y.

    (c) Solve the equations and find out:

        (i)   how many people were on the coach;

        (ii) how much less the ride would have cost if only the children had gone on it.

4. (a) I think of a number. I divide by 5, then add 7 and finally multiply by 3. If my final answer is the same as the number I started with, then what is this number?

(b) I think of a number. I add ten to it. I multiply the result by three. I take this result away from 114, and divide the answer by three. Finally, I add the original number to this answer. Show that, whatever number I start with, my final answer is always the same number.

(c) I think of a number. I divide by 5, then add 7, then multiply by 4. The number resulting is increased by 25% to get a final answer. What *single* calculation would give this final answer from my original number?

5. *[For this question, you may assume that volumes are additive: for example, mixing 20ml of ethanol and 80ml of another liquid makes 100ml of liquid.]*

In the chemistry laboratory, I have three very large containers of different liquids; each liquid has a certain concentration of ethanol. By volume, liquid A contains 40% ethanol, liquid B contains 5% ethanol, and liquid C contains 13.5% ethanol. I mix in a fourth container 1 litre of liquid A, 3 litres of liquid B and 6 litres of liquid C.

(a) Show that the percentage of ethanol in the new mixture is 13.6%.

I then continue to add more of liquid A to the mixture until it has a concentration of 35% ethanol.

(b) What is the total volume of this mixture?

6. (a) A triangle is isosceles and right-angled. A side, which is not the hypotenuse, has length $\sqrt{2}$ cm. Find the length of the hypotenuse.

(b) A regular octagon *ABCDEFGH* has side length 2cm. The sections created by adding line segments AF, BE, CH and DG are shaded black or white as shown. Find the following ratio:

white area : black area

Leave your answer in terms of $\sqrt{2}$ in its simplest form.
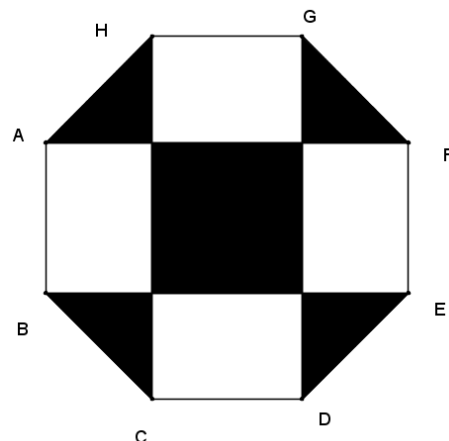
7.   (a)   Expand the brackets $(x-y)^2$ and simplify.

         (b)   Simplify $\dfrac{x}{xy} - \dfrac{y}{xy}$.

         (c)   There are two numbers x and y (neither of which is a whole number) such that $x-y=2$ and $xy=9$. WITHOUT trying to find either x or y, find the values of:

                 (i) $\dfrac{1}{y} - \dfrac{1}{x}$

                 (ii) $x^2 + y^2$

                 (iii) $\dfrac{1}{x^2} + \dfrac{1}{y^2}$

8.   (a)   I have two fair dice; they each have six sides numbered 1, 2, 3, 4, 5 and 6. I roll them and add the two numbers together. I work out the probability of getting a score of 12 using the following argument:

            o   the smallest score is a two

            o   the highest score is a twelve

            o   there are therefore 11 possible different scores

            o   therefore the probability of a 12 is $\frac{1}{11}$

         Without calculating the *actual* probability, explain why this argument is wrong, and explain whether the actual probability should be less than or more than $\frac{1}{11}$.

         (b)   A mustard seed is dropped from a great height so it lands at random onto a carpet with a pattern of black circles each touching 4 others (part of which is shown) spreading out in all directions. If the seed is far smaller than each circle, find the probability the seed lands in a white section. Give your answer to 3 significant figures.
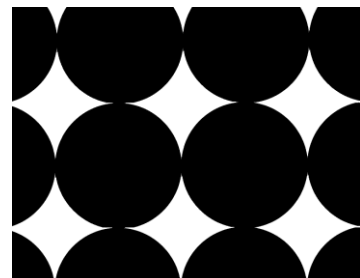
9. *In this question, diagrams are not drawn to scale.*

(a) In the triangle shown, the following are true:

- $A\hat{B}C = 2x + 12y$ degrees

- $B\hat{C}A = 8x + 5y$ degrees
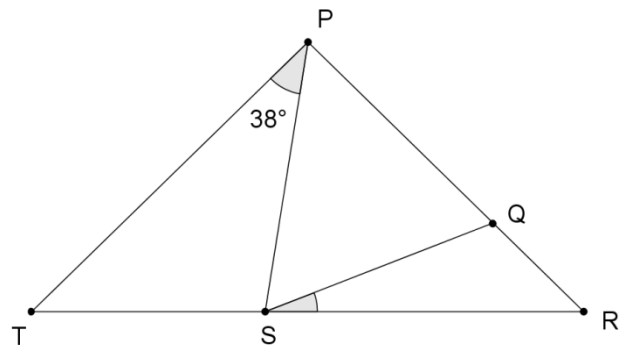
- $C\hat{A}B = 14x + 15y$ degrees

- $AB = AC$.
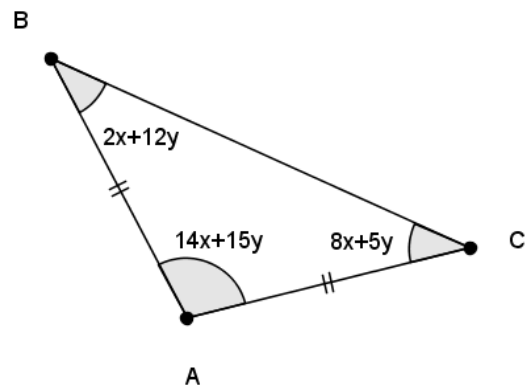
Find:

(i) x

(ii) $A\hat{B}C$

(b) In the triangle shown, $T\hat{P}S = 38°$, side $TP = PR$ and side $PS = PQ$.

Find angle $Q\hat{S}R$.

10. (a) There are two positive, whole numbers, x and y, such that x is bigger than y and also $7x - 5y = 20$. Find both x and y.

(b) I pick out all the coins in my pocket and discover I have 30 coins, of total value £1.70. I only have two-pence, five-pence and ten-pence coins, and I have more ten-pence coins than I have two-pence coins.

(i) How many five-pence coins do I have?

(ii) Show there is only one solution to the problem.

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