Eton College King's Scholarship Examination 2014

(One and a half hours)

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

*Show all of your working. The use of calculators is **not** permitted.*

Do not turn over until told to do so.

1. This question is compulsory.

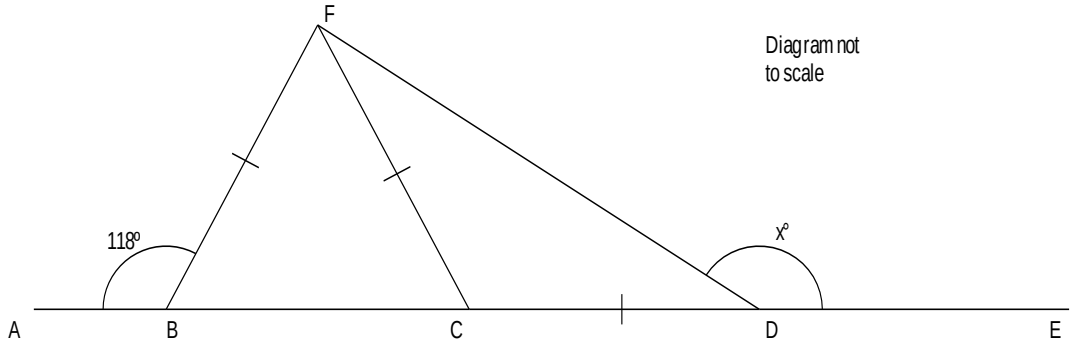
(a) Calculate the following, leaving your answers as simplified fractions:

(i) $4\frac{1}{12}+5\frac{5}{48}$ [3]

(ii) $\frac{34}{35}\times\frac{42}{51}$ [3]

(b) Calculate 115% of 80% of £15. [3]

(c) In the diagram below, ABCDE is a straight line and all three lengths BF, CF and CD are equal. Given that angle ABF = 118°, calculate the angle EDF (marked as x^o in the diagram). [2]



(d) What do I subtract from $4-5x$ to get $-4x+5$? [3]

(e) Solve the following equations, giving your answers as simplified fractions:

(i) $\frac{3}{28}(a+5)=\frac{5}{8}$ [3]

(ii) $1-\frac{1}{3}b=b-1$ [3]

(f) Solve the simultaneous equations:

$$11x-8y=18$$
$$7x+4y=16$$

[4]

(g) Solve the following inequalities:

(i) $\frac{2}{3}x>1+\frac{1}{2}x$ [3]

(ii) $2x-7\geq 2+7x$ [3]

(h) Two circles have radii 0.4 cm and 1.2 cm. The area of the smaller circle is x and the total area of the two circles is y. What percentage of y is x? [4]

(i) The diagram shows a triangle ABC. The point P lies on the side BC and angle CPA is 90^o. AB and AP are 4 cm and 2.4 cm respectively. PC is 1 cm.

(i) Show that AC has length 2.6 cm. [3]

(ii) Calculate the perimeter of the triangle ABC. [4]

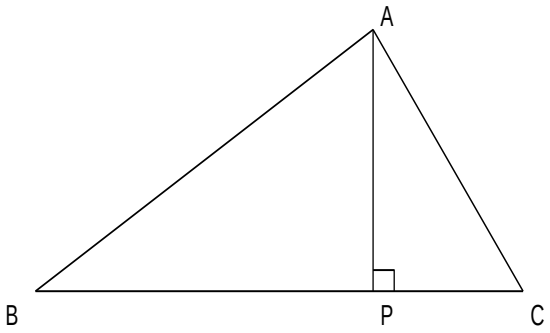


Diagram not to scale

(j) Edward scores an average of 26 runs in his first three cricket matches. After his first five cricket matches, he scores an average of 27.8 runs. If he scored 9 more runs in his fifth match than he did in his fourth match, how many runs did he score in his fifth match? [4]

(k) (i) By what do you multiply $5\frac{1}{4}$ to get 7? Give your answer as an exact fraction. [2]

(ii) By what do you multiply $\frac{a}{2b}$ to get $\frac{ab}{4}$? [3]

2. (a) Show that $(a+2b)^2 = a^2 + 4ab + 4b^2$. [2]

(b) (i) Multiply out and simplify the expression $(a+2b)^2 + (2a-b)^2$. [3]

(ii) Hence calculate $1002^2 + 1999^2$. [2]

(c) Find two whole numbers *n* and *m* such that $n^2 + m^2 = 20000245$. [3]

3. (a) In the diagram below, the lines AB and ED are parallel.
Angles ABC, BCD and CDE are x^o, y^o and z^o respectively.

Giving reasons, show that $x - y - z + 180 = 0$. [4]

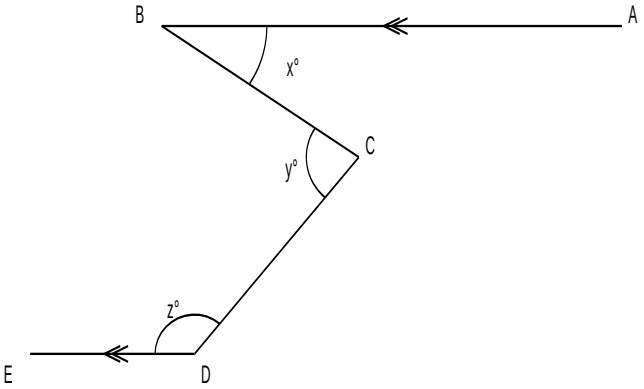


Diagram not to scale

(b) The diagram below shows a parallelogram PQRS.
The two lengths PX and QX are equal.
The two angles SRX and QRX are equal.

(a) Giving reasons, show that triangle XQR is isosceles. [2]

(b) Giving reasons, show that angle SXR is a right angle. [4]

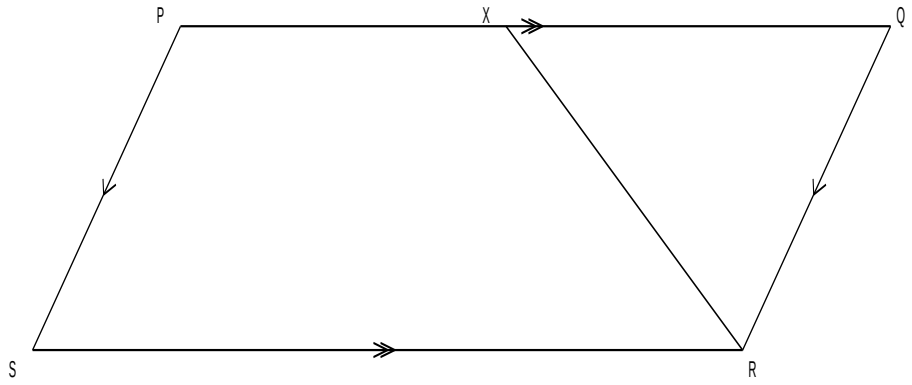


Diagram not to scale

4. You are given that $123456789 = 3803 \times 32463$.

(a) Write down the answers to the following calculations:

(i) 0.3803×32463000 [1]

(ii) $\dfrac{12345.6789}{380300}$ [2]

(iii) $\dfrac{12345.6789}{0.03803}$ [2]

(iv) $\dfrac{3803123456789}{3803}$ [2]

(b) Calculate the value of $\dfrac{38033803 \times 3246332463}{123456789}$. [3]

5. In this question, a word is defined to be a set of letters, each letter of which is either an L or an R, written down in order. A word is good if it does not contain 3 consecutive letters which are the same. Otherwise a word is bad. The length of a word is the number of letters in it.

For example, LRRL is a good word of length 4 and is a different good word to LRLR. LRRRL and LRLLLLR are both bad words.

(a) Write down all the good words of length 4. [2]

(b) If I write down a good word which ends in LR, explain why I can make two different good words from it by adding a letter at the end. Would this be true if I started with a good word ending in LL?

[2]

(c) Copy and complete the following table.

	Number of good words of length 4	Number of good words of length 5	Number of good words of length 6
Last two letters LL	2		
Last two letters RL	3		8
Last two letters RR	2		
Last two letters LR	3		

[2]

(e) How many good words are there of length 15?

[4]

6. The number 12 has 6 factors (1, 2, 3, 4, 6 and 12).

(a) Find the number of factors (you need not list them) for each of the following:
(i) 75
(ii) 847 [2]

(b) Explain clearly why $7^4 \times 11^3$ has exactly 20 factors. [2]

(c) How many factors does $2^4 \times 3^2 \times 5 \times 7^3$ have? [2]

(d) Explain why all square numbers have an odd number of factors. [2]

(e) Find the smallest number with exactly 36 factors. [2]

[End of paper]