

# Eton College King's Scholarship Examination 2013

## MATHEMATICS B (One and a half hours)

- *Answer as many questions as you can.*
- *Each of the ten questions carries ten marks.*
- *Show all of your working.*
- *Calculators are permitted and some questions will require them.*
- *Answers should be given to three significant figures unless they can be given exactly or the question states otherwise.*

1. In this question, you should use the  $\pi$  button on your calculator.
  - a) A circle C has a radius of 25 cm.
    - i) Find the area of the circle correct to 5 significant figures.  
A square S has the same area as the circle C.
    - ii) Find the perimeter of the square.
  - b) A square T has a side length of 10cm, and a circle K has an area which is twice that of the square T. Find the circumference of the circle K.
  
2.
  - a) Boys from two classes take a maths test. Class A has 18 boys in it, and Class B has 25 boys in it. The mean score of Class A is 67.5%; the mean score of Class B is 87.2%. Find the mean score of all the boys correct to 3 significant figures.
  - b) I cycle 30 km at 18.4 km/hr then jog 30 km at 5.8 km/hr. Find my average speed for the whole journey, correct to 4 significant figures.
  
3. Let S represent the set of all whole numbers between 1 and 5000 inclusive.
  - a) How many numbers in S are divisible by 3?
  - b) How many numbers in S are divisible by 7?
  - c) Show that 2142 numbers in S are divisible by 3 or 7 or both.
  - d) How many numbers in S are divisible by 3, 7 or both but not by 13?
  
4. In Lo-Maqom, salaries are paid in Lo-Maqomian pounds. Tax is collected annually in the following manner:
  - on the first £5,000 earned, no tax is paid;
  - on the next £5,000 earned, 25% tax is paid;
  - on the next £10,000 earned, 40% tax is paid;
  - on any further income, 60% tax is paid.
  - a) Show that a plumber earning £90,000 pays £47,250 tax.
  - b) Calculate how much tax is paid by a lawyer earning £15,000.
  - c) A construction worker earns £67,500. What percentage of her salary does she pay in tax?
  - d) A sailor pays 33% of his salary in tax. What is his salary?

[Turn over]

5. a) List the prime numbers between 50 and 70.
- b) Find which of these prime numbers divides 2013 and hence find the full factorization of 2013.
- c) Multiply out and simplify  $(x - y)(x + y)$ .
- d) Two whole numbers, which differ by no more than 10, have squares which differ by 2013. Using your results from parts (c) and (b), find the smaller of these numbers.  
*[Working must be shown; no credit will be given for answers found by trial and error]*
6. In Eton General Stores, they have just started to stock cats, mats and hats; all cats are the same price as each other, all hats are the same price as each other, and all mats are the same price as each other.
- In the Store, Ashley pays £125 for a cat and a hat, whilst Bernard pays £94 for a hat and a mat.
- a) How much more is a cat than a mat?  
 Charles claims he bought 10 cats and 9 mats for £690.
- b) Assuming this is true, how much is a cat?  
 Charles is a notorious liar, and it wasn't true. Derek, who is truthful but forgetful, says he can't remember exactly how much he paid for 4 mats and a hat on Monday, or how much he paid on Tuesday for 5 cats, but he remembers he paid £145 more on Tuesday than on Monday.
- c) Find the actual price of a cat.
7. In this question, the letters  $n$  and  $m$  represent whole numbers.
- a) For each expression, write down whether it is *always* divisible by 3, *never* divisible by 3, or *sometimes* divisible by 3. In each case, give a reason:
- i)  $21n^2$
- ii)  $33n + 15m$
- iii)  $5n + 7m$
- iv)  $9m + 6n + 13$
- b) Multiply out and simplify:
- i)  $(3n + 1)(3n + 1)$
- ii)  $(3n - 1)(3n - 1)$
- iii)  $(3n)^2$
- c) Explain clearly why none of the square numbers (1, 4, 9, 16, ...) is one less than a multiple of 3.

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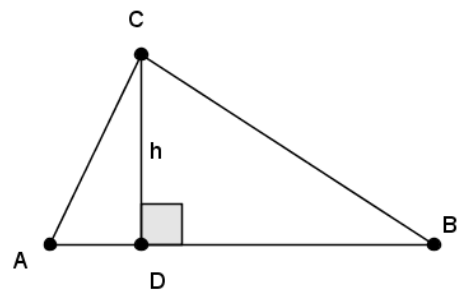
8. Four different pumps A, B, C and D can be used individually or together to fill up identically sized oil tankers; each pump works at its own constant rate. Pump A alone will fill a tanker in 30 minutes and pump B alone fill a tanker in 20 minutes.

a) If both pump A and pump B are used together, show that it will take 12 minutes to fill a tanker.

Pump C alone fills a tanker in 3 hours, and pumps A, C and D together fill a tank in 21 minutes and 36 seconds.

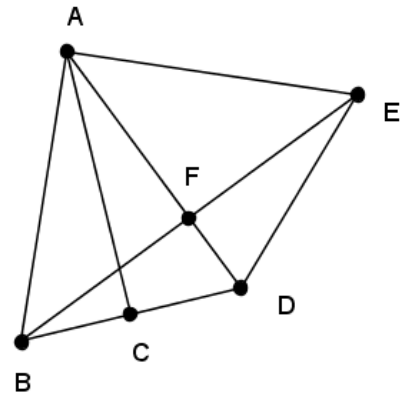
b) How long would it take pump D alone to fill a tanker, in hours and minutes?

9. a) In the diagram on the right, the triangle  $ABC$  has an area of  $314 \text{ cm}^2$ . The line  $CD$  has length  $h \text{ cm}$  and is perpendicular to the line  $AB$ , which has length  $112 \text{ cm}$ . Find  $h$  to 3 significant figures.



- b) In the diagram below right, triangles  $ABD$  and  $AED$  are both isosceles; the angles  $\hat{A}BD, \hat{A}DB, \hat{A}DE$  and  $\hat{A}ED$  are all equal, the length  $AB$  is  $7225 \text{ mm}$  and the length  $DE$  is  $6120 \text{ mm}$ . The point  $C$  lies on  $BD$  equidistant from  $B$  and  $D$ ; the line  $BE$  intersects  $AD$  at the point  $F$ .

- i) Find  $AC$ .
- ii) Find the area of triangle  $ABD$ .
- iii) Write down the size of angle  $\hat{A}FB$ .
- iv) Find the length  $BE$ .



[Turn over]

10. Consider the following expression for  $S$ , involving consecutive multiples of 7:

$$S = 7 + 14 + 21 + 28 + \dots + 1386 + 1393 + 1400$$

- a) How many numbers are being added together?

Because all the numbers are being added, they can be re-ordered in the following way:

$$S = (7 + 1400) + (14 + 1393) + (21 + 1386) + \dots + (693 + 714) + (700 + 707)$$

- b) Calculate each of the five brackets shown, and write down what pattern you notice.
- c) Explain clearly why this pattern should hold for the remaining brackets which are not shown.
- d) Hence show that  $S = 140700$
- e) Use a similar method to find
- i)  $T = 3 + 6 + 9 + 12 + \dots + 2994 + 2997 + 3000$
  - ii)  $U = 9 + 13 + 17 + 21 + \dots + 2009 + 2013$
  - iii)  $V = 3 + 8 + 13 + 18 + \dots + 2008 + 2013$

**[END OF PAPER]**

**[Turn over]**