Eton College King's Scholarship Examination 2013

(One and a half hours)

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

*Show all of your working. The use of calculators is **not** permitted.*

1. This question is compulsory.

(a) Find the mean average of the numbers 1.8, 3.5 and 7.3. [2]

(b) Given that $a = -2$ and $b = 3$, calculate the exact value of $b^2 - ab - a^2$. [3]

(c) Calculate the following, leaving your answers as mixed numbers:

(i) $3\frac{2}{5} + 2\frac{1}{4}$ [3]

(ii) $\frac{44}{7} \times \frac{98}{121}$ [3]

(d) What do you add to $5x + 7$ to get $2x + 11$? [2]

(e) Solve the following equations, leaving your answers as exact fractions:

(i) $\frac{2}{3}(a-1) = \frac{4}{15}$ [3]

(ii) $4b + \frac{2b}{5} = 11$ [3]

(f) The value of my car will decrease by 15% during the period from May 2013 to May 2014.

(i) If it is worth £23,000 in May 2013, by how much will its value have decreased by May 2014?

[2]

(ii) The value of the car increased by 15% during the period from May 2012 to May 2013. Will the car be worth more, less or the same in May 2014 as it was in May 2012? Justify your answer.

[3]

(g) Solve the simultaneous equations:

$$5x + 3y = 12$$
$$y = 2x - 7$$

[4]

(h) Roland wishes to hire a drill. From the supplier he usually uses, the total cost is a fixed cost of £45 plus an extra cost of £4.00 per day. A rival supplier offers him a fixed cost of £36 plus an extra cost of £4.45 per day.

(i) Show that if Roland is hiring the drill for 12 days, it is £3.60 cheaper for him to hire it from the rival supplier.

[4]

(ii) Let d be the least number of days for which it is cheaper for Roland to hire the drill from his usual supplier. Find d.

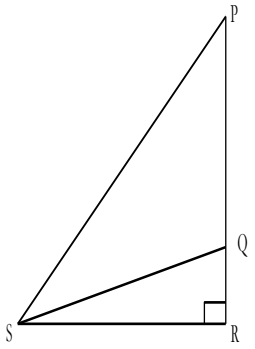
[3]

(i) Solve the following inequalities:

(i) $-3x < 18$ [2]

(ii) $1 - \frac{x}{2} < 2x$ [3]

(j) In the diagram below the points P, Q, and R all lie on a straight line and angle PRS is a right angle. The lengths PS and PR are 17 cm and 15 cm respectively.



(i) Find the length SR. [3]

(ii) Given that the length PQ is 13.2 cm, show that the length SQ is 8.2 cm. [3]

(k) (i) By what do you multiply $6\frac{1}{4}$ to get 5? Give your answer as an exact fraction. [2]

(ii) By what do you multiply $\frac{1}{2}p^2$ to get pq? [2]

2. Consider the product

$$\frac{4}{4-n} \times \frac{5}{5-n} \times \frac{6}{6-n} \times \ldots\ldots \times \frac{21}{21-n}$$

where n is an whole number excluding 4, 5, 6, ……21.

(a) Show that when $n = 1$, the product is equal to 7, and find the product when $n = 2$. [4]

(b) Evaluate the product when

(i) $n = 25$;

(ii) $n = 24$;

(iii) $n = 23$. [6]

3. In this question, the diagrams are not drawn to scale.

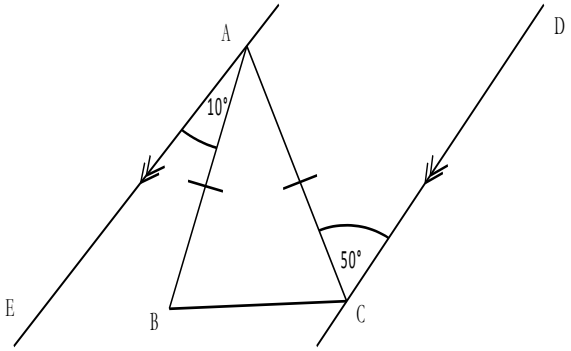
(a) In the diagram below, the lines AE and DC are parallel. The lengths AB and AC in the triangle ABC are equal. Furthermore angles EAB and ACD are 10^o and 50^o respectively.

(i) Giving a reason, write down the angle EAC.

[2]

(ii) Show that the angle ACB is equal to 70^o.

[3]



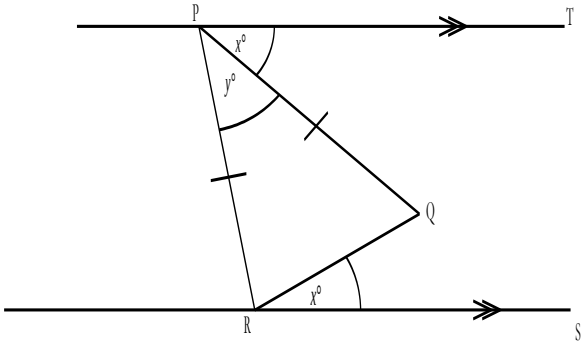
(b) In the diagram below, the lines PT and RS are parallel. The lengths PQ and PR in the triangle PQR are equal. Both angles TPQ and QRS are equal to x^o and angle QPR is equal to y^o.

(i) Find an expression for y in terms of x. You should give clear reasons and leave your answer as simply as you can.

[4]

(ii) Explain why x^o must be smaller than 45^o.

[1]



4. (a) Solve the following equations:

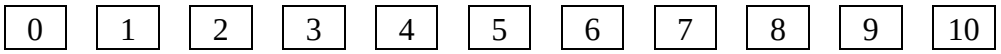
(i) $14-\frac{x}{5}=8-\frac{x}{11}$ [3]

(ii) $1-\frac{x}{5}=3\left(1-\frac{x}{3}\right)$ [3]

(b) Two aeroplanes have the same size fuel tanks, but one will travel for 21 hours on a full tank and the other for 35 hours on a full tank. They set off at the same time with full tanks. After how long will one have three times as much fuel left as the other?

[4]

5. The diagram below shows 11 stepping stones all in a row. Oliver starts by standing on stone 0 and wants to reach stone 10. He does this by making a series of jumps, each of which allows him to advance towards stone 10 by either one or two stones. He lists the jumps he makes in order.



For example, if he produces the list 212212, it means he jumped from stone 0 to stone 2, then stone 2 to stone 3, then stone 3 to stone 5, then stone 5 to stone 7, then stone 7 to stone 8, and then stone 8 to stone 10.

Furthermore he would count the list 221212 as different from one above, as it means that he has landed on different stones during his path from stone 0 to stone 10.

(a) If Oliver starts at stone 0 and only reaches stone 4, write down the 5 different lists of 1s and 2s that could represent the series of jumps he could have taken.

[2]

(b) Explain why to get to stone 5, Oliver must have jumped onto either stone 3 or stone 4.

[1]

(c) Use your answers to (a) and (b) to show there are 8 different lists of 1s and 2s that could represent the series of jumps he could have taken to start from stone 0 and reach stone 5.

[2]

(d) Explain why to get to stone 6, Oliver must have jumped onto either stone 4 or stone 5. Hence show that there are 13 different lists of 1s and 2s that could represent the series of jumps Oliver could have taken to start from stone 0 and reach stone 6.

[2]

(e) Find the number of different lists of 1s and 2s that could represent the series of jumps Oliver could have taken to start from stone 0 and reach stone 10.

[2]

6. For any two positive whole numbers n and m, $n*m$ is defined to be the lowest common multiple of n and m divided by the highest common factor of m and n.
For example,

the lowest common multiple of 12 and 8 is 24;
the highest common factor of 12 and 8 is 4;

thus $12*8 = \frac{24}{4} = 6$.

(a) Calculate
(i) $6*36$
(ii) $6*35$
(iii) $15*25$ [3]

(b) If n is a whole number, express the following in terms of n:
(i) $n*n$
(ii) n^2*n [2]

(c) Suppose that a is a whole number such that $a*6 = 1$. Explain why a can only be 6. [2]

(d) Suppose that b is a whole number such that $b*6 = 6$. Find, with justification, all the possible values of b. [3]

[END OF PAPER]