

# Eton College King's Scholarship Examination 2012

(One and a half hours)

## MATHEMATICS A

*Answer Question 1 and as many of the other five questions as you can.*

*Question 1 is worth 50 marks. All other questions are worth 10 marks each.*

*Show all of your working. The use of calculators is permitted.*

1. This question is compulsory.

(a) If  $x = -5$  and  $y = 12$ , evaluate the following, leaving your answers as exact fractions:

(i)  $\frac{8x^2 - y^2}{y - 12x}$  [3]

(ii)  $\frac{y}{2x} + \frac{x}{y}$  [2]

(b) Solve the following inequalities:

(i)  $\frac{2}{3}(x - 3) < 18$  [2]

(ii)  $8 - 3x < 2x - 2$  [2]

(c) Simon and Terry are both told to draw an isosceles triangle which has two angles differing by  $15^\circ$ . They both draw a triangle but find they have drawn ones with different angles from each other. Can they both be correct? [3]

(d) Calculate

(i) 15% of £40.00 [1]

(ii) 89% of £111.00 [1]

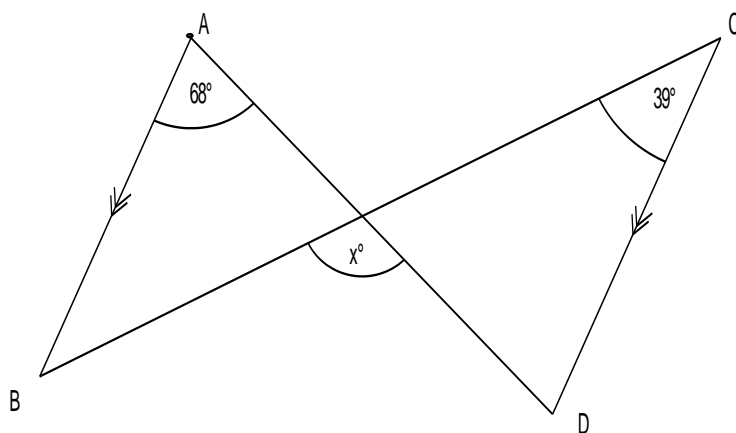
(e) A triangle of base length 29.7 cm has area  $8.9 \text{ cm}^2$ . Find the height of the triangle, giving your answer correct to 2 significant figures. [3]

(f) Solve the following equations, leaving your answers as mixed numbers where appropriate:

(i)  $\frac{4 - 3x}{5} = 9$  [2]

(ii)  $\frac{2 - 5x}{3} = \frac{5 - 3x}{2}$  [3]

(g) In the diagram below, AB and CD are parallel. Calculate the value of  $x$ . [2]



(h) Solve the following simultaneous equations

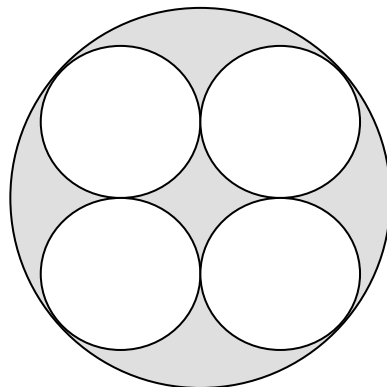
$$4x - 3y = 15$$

$$5x + 7y = 8$$

[4]

- (i) Simplify the following as far as possible:
- (i)  $(3ab^4)^3$  [2]
- (ii)  $\frac{8d^2 - 3d^2}{20d^2}$  [2]
- (j) A rectangular field has length 30 m. Its width is half its length.
- (i) Find, correct to 3 significant figures, the distance between the opposite corners of the field. [3]
- (ii) A second field is 9% less wide but 9% longer. Find the distance, correct to 3 significant figures, between the opposite corners of the second field. [3]
- (k) (i) By what do you multiply 8 to get  $2\frac{1}{2}$ ? Give your answer as an exact fraction. [2]
- (ii) By what do you multiply  $ab$  to get  $\frac{2b^2}{3}$ ? [2]
- (l) A large company insists that each shareholder invests at least £12,000 in the company. At present, the company has two thousand shareholders and their average investment is £13,040.
- (i) How much is their total investment? [2]
- (ii) Suppose that 100 new people become shareholders in the company. What is the lowest level to which the average investment could drop? Give your answer to the nearest pound. [2]
- (iii) In fact a further  $n$  new people become shareholders and on average they invest £12,320. If the average investment across all the shareholders is now £12,960, find  $n$ . [4]

2. The diagram shows four identical circles inside one large circle. The radius of each of the smaller circles is  $\sqrt{2}$  cm.



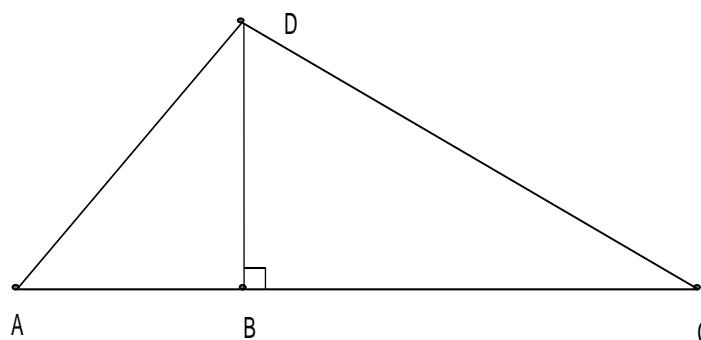
- (a) Prove that the radius of the large circle is  $2 + \sqrt{2}$  cm. [3]
- (b) Show that the shaded area is  $2\pi(2\sqrt{2} - 1)$  cm<sup>2</sup>. [4]
- (c) Find an exact expression for the total perimeter of the shaded area. [3]

3. This question is about factors of numbers.

- (a) Write down all of the 6 factors of 45. [2]
- (b) Two whole numbers multiply to give 45. Explain why their sum must be even. [1]
- (c) Two whole numbers multiply to give 32. Explain why if their sum is odd, the two numbers must add to 33. [2]
- (d) Two whole numbers multiply to give 81. What are the possible values for their sum? [2]
- (e) Two whole numbers multiply to give 1,417,176 and add to make 354,298. Find the two numbers. [3]

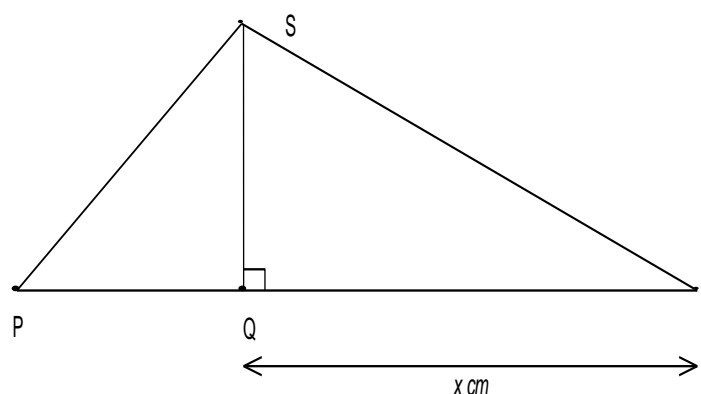
4. In this question, the diagrams are not drawn to scale.

- (a) In the diagram below, the points A, B and C all lie on a straight line and the angle CBD is a right angle. AD and CD are 3 cm and 8.2 cm respectively and AB is 2.4cm.



- (i) Calculate the length BD.
  - (ii) Calculate the length BC. [4]
- (b) Show that  $(c - x)^2 = c^2 - 2cx + x^2$ . [2]
- (c) In the diagram below, P, Q and R lie on a straight line and angle SQR is a right angle. The lengths PS, SR and RP are  $a$ ,  $b$  and  $c$  cm respectively and QR is  $x$  cm. Use algebra to show that

$$x = \frac{b^2 + c^2 - a^2}{2c} \quad [4]$$



5. (a) A postman has ten letters to deliver. Each letter is addressed to exactly one of nine addresses. Explain why at least two letters have the same address on them. [1]
- (b) 230 Etonians have their birthday during the week starting Monday 30th July. Explain why at least 33 of them must have their birthday on the same day. [3]
- (c) At a birthday party, thirty five sweets are shared between eight children. Given that each child receives at least one sweet, is it possible for them all to receive a different number of sweets? [3]
- (d) A rectangle has width 6 cm and height 12 cm. If 9 points are chosen from within the rectangle, explain why two of the points must be at most  $\sqrt{18}$  cm away from each other. [Hint: divide the rectangle into squares of equal area.] [3]
6. (a) Show by multiplying out that if  $(x - A)(y - A) \leq 0$ , then
- $$xy \leq A(x + y - A) \quad [2]$$

For any set of numbers, the R algorithm is as follows:

- Work out the mean of the numbers: call the answer  $A$
- Replace  $x$ , the smallest number in the set, and  $y$ , the largest number in the set, by  $A$  and  $x + y - A$  respectively. (If there is more than one smallest number, replace the first. If there is more than one greatest number, replace the first.)

For example, for the number set  $\{1, 4, 6, 9\}$ ,  $A = 5$  and so 1 is replaced by 5 and 9 is replaced by 5. Thus the R algorithm replaces  $\{1, 4, 6, 9\}$  with  $\{5, 4, 6, 5\}$ .

Furthermore, the R algorithm replaces  $\{5, 4, 6, 5\}$  by  $\{5, 5, 5, 5\}$ .

- (b) Show that if the R algorithm is applied twice to  $\{10, 13, 23, 29, 35\}$ , the resulting set of numbers is  $\{22, 22, 23, 20, 23\}$ . [2]
- (c) Show further that if the R algorithm is applied 4 times to  $\{10, 13, 23, 29, 35\}$ , the resulting set of numbers is  $\{22, 22, 22, 22, 22\}$ . [2]
- (d) Use part (a) to explain why when you apply the R algorithm to a set of numbers, then the new set of numbers will not multiply to give a smaller answer than the original set of numbers. [2]
- (e) Explain why if you repeat the R algorithm,  $A$  remains unchanged. [1]
- (f) Use your answers to the earlier parts to explain why  $10 \times 13 \times 23 \times 29 \times 35 \leq 22^5$ . [1]

**[END OF PAPER]**