

Eton College King's Scholarship Examination 2011

MATHEMATICS B

(One and a half hours)

- *Answer as many questions as you can.*
- *Each of the ten questions carries ten marks.*
- *Show all your working.*
- ***Calculators are not allowed.***

1. a) Show that 1001 is divisible by 13 and hence write 1001 as a product of its prime factors.

- b) Simplify the following fractions:

i) $\frac{1111}{11}$

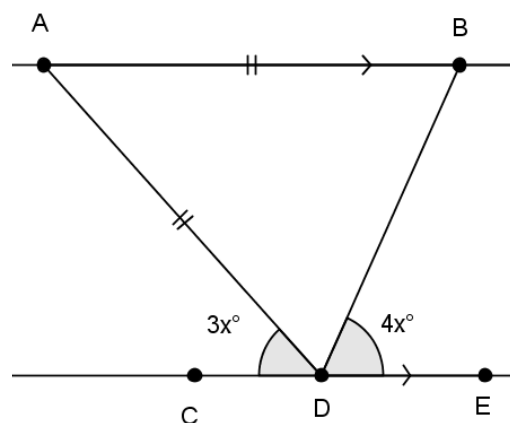
ii) $\frac{333333}{111}$

- c) Evaluate the following, giving your answer in its simplest form:

$$\frac{777777}{26} \times \frac{10}{111} \times \frac{1}{49}$$

2. a) Solve the equation $\frac{3}{4}x - \frac{1}{3}(2 - x) = x$

- b) The diagram shows an isosceles triangle ABD where $AB = AD$. The point D lies on the line CE , which is parallel to the line AB . Angle $CDA = 3x^\circ$ and angle $EDB = 4x^\circ$. By considering all the angles in triangle ABD , find x as a mixed fraction.



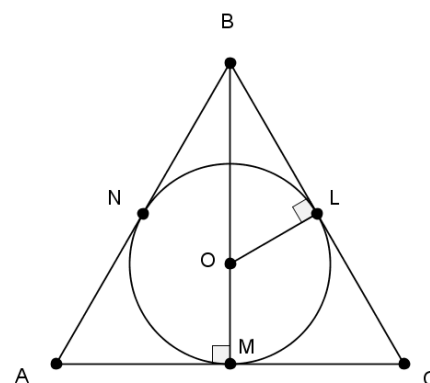
3. a) If x bottlefuls of water are required to fill a tank in the shape of a cube with side length y cm, how many bottlefuls are required to fill a tank in the shape of a cube with side length z cm?
- b) One day, it takes p hours for q factory workers to construct r items. The following day, three more workers join the team and four more items are required to be constructed.
- (i) Find how many hours it will take, in terms of p , q and r .
- (ii) If in fact it takes the same length of time, find an equation and show that it reduces to $4q = 3r$

4. At a pet shop, I buy a dog and a frog for a total of £80. I then sell the dog at a 20% profit and the frog at a 25% profit, and make a total profit of £19.

- a) If I bought the dog for d pounds, show that I sold it for $1.2d$ pounds.
- b) By finding and solving two equations simultaneously, find the price for which:
- (i) I bought the frog;
- (ii) I sold the dog.

5. a) A man drives down a long straight road. For the first 20 minutes, he drives at 45 km/hr but for the next 5 kilometres he drives at 75 km/hr. Find the following:
- the distance, in kilometres, travelled at the slower speed;
 - the time, in minutes, spent travelling at the faster speed;
 - the average speed of the journey, in kilometres per hour.
- b) 150 people in a village all voted in an election with four candidates. Mr Green got a third of the number of votes that Mr Yellow got, but Mr Blue got twice as many votes as Mr Yellow. Mr Red got 40 more votes than Mr Green.
- Let y represent the number of votes that Mr Yellow received.
- Write down how many votes Mr Red received, in terms of y .
 - Hence find out how many votes Mr Yellow and Mr Blue received.

6. A shape is made of a circle, with centre O , inscribed in an equilateral triangle ABC , touching it at the points L , M and N , with lines OL and BOM added as shown. The angle $\hat{OLB}$ and $\hat{OMA}$ are both right angles, and the length $AM = \sqrt{27}$ cm

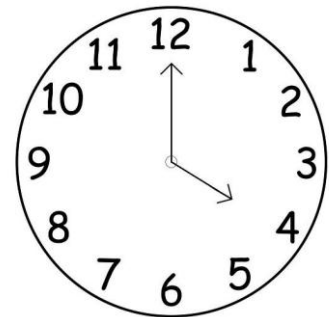


- Using only the *fewest* lines in each case, which lines must be added so that the shape has:
 - reflectional symmetry in the line BOM ;
 - reflectional symmetry in the line OL ;
 - rotational symmetry of order 3 in the point O .
 - Write down the length of CM .
 - Write down the length of BC .
 - By considering triangle BCM , find the length BM .
 - Let the radius of the circle be r cm. Find the length OB in terms of r .
 - By considering triangle OLB , find the radius of the circle.
7. a) This year, 1st September will be a Thursday.
- What day of the week will September 30th be?
 - How many Mondays will there be in September?
 - How many Mondays will there be in August?
 - How many Mondays will there be in August 2012? Show your working.
- b) In a particular month one year, there were an odd number of Tuesdays that fell on odd-numbered days of the month. What are the only possible days of the week that the 18th of the month could have been? Show your working.

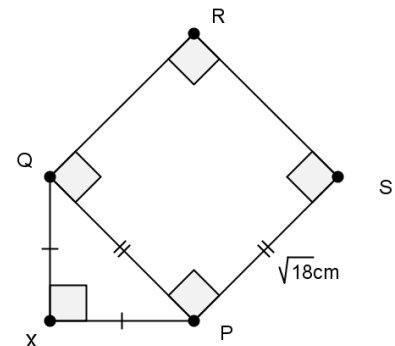
8. 200 pupils attend Eton Sports Academy. In winter, each pupil must play either the Wall Game or the Field Game (but not both) and in summer, each must row or play cricket (but not both). 30% of the pupils play the Field Game; 40% of those who play the Wall Game also row; $\frac{2}{3}$ of those who play cricket also play the Wall Game.
- How many play both cricket and the Wall Game?
 - How many both row and play the Field Game?
 - What fraction of the rowers also play the Wall Game? Simplify your fraction.

9. At 4 o'clock and at 8 o'clock, the angle between the hands of a clock is 120 degrees.

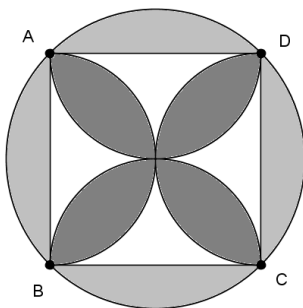
- Find the angle between the hands:
 - at 10 o'clock;
 - at half past five;
 - at 3:15.
- How many times are the hands pointing in the same direction between 1am on one day and 1am the following day?
- How many times are the hands at right angles to one another between 1am on a Sunday and 1am on a Sunday a fortnight later?



10. a) What is the angle between the two diagonals of a square?
- b) As shown in the diagram, a square $PQRS$ has sides of length $\sqrt{18}$ cm, and side PQ forms the hypotenuse of an isosceles right-angled triangle PQX .
- State the angles in the triangle PQX .
 - Find the length PX .
 - Find the area of the pentagon $PXQRS$.



- c) As shown in the diagram, a square is inscribed inside a circle of radius 3cm. Within the square, four semi-circular arcs are drawn, with diameters AB , BC , CD and DA . The area enclosed between the square and the circle is shaded lightly, and the area between the semi-circles is shaded darkly.



Show that the lightly-shaded area and the darkly-shaded area are equal.

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