Eton College King's Scholarship Examination 2009

MATHEMATICS B   **(One and a half hours)**

Answer as many questions as you can. Each of the ten questions carries ten marks. Show all your working. Calculators are not allowed.

1.   a)   If a car can travel x kilometres on y litres of petrol, how far can it travel on z litres of petrol?

       b)   A car travels 15 kilometres per litre of petrol. How much petrol would be used if it travelled for one and a half hours at a speed of 84 kilometres per hour?

       c)   If 16 officials can count 6120 ballot papers in $1\frac{1}{4}$ hours, how long will 40 officials take to count 153000 papers, working at the same rate?

2.   a)   To cover a distance of 10 km, a girl runs some of the way at 15 km/h, and walks the rest of the way at 5 km/h. Her total time for the journey is 1 hour.
            i)   If the distance walked is x km, find a simple expression for the distance run.
            ii)   Derive an equation involving x and use it to find the distance walked.

       b)   At a certain fast food restaurant, one can buy 3 burgers, 7 drinks and one order of fries for £12. At the same restaurant, 4 burgers, 10 drinks and one order of fries cost £16.45. How much would an ordinary meal of one burger, one drink and one order of fries cost?

3.   a)   Expand and simplify $(m+n)(m-n)$.

       b)   Find all the factors of 273.

       c)   We wish to find pairs of perfect square numbers that differ by 273.
One such case is $137^2 - 136^2 = 273$. Use your answers to parts (a) and (b) to help you find the other three.

4.   a)   If the hypotenuse of a right-angled triangle has length 2cm and short side 1.2cm, calculate the length of the third side.

   b)   The irregular pentagon shown has three right angles and side lengths labelled.

           Calculate the length x cm.
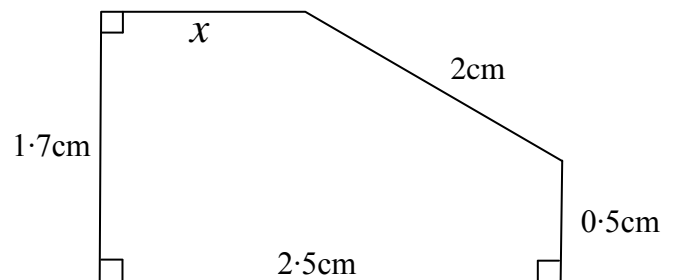
       c)   A ladder is resting against a vertical wall such that its foot is 1.4m away from the base of the wall. When a boy steps on the ladder, it slips down the wall 0.8m and its foot is now 3m away from the wall. Calculate the length of the ladder.

5. a) Solve the equation $\frac{1}{2}(x-4)=3-\frac{2}{3}(6-x)$.

 b) A man is 6 years older than his wife. He observed that the difference between two-thirds of his wife's age in ten years' time and half his age four years ago is fourteen years. Write down two equations involving the current ages of the man and his wife and hence solve them to find their current ages.

6.
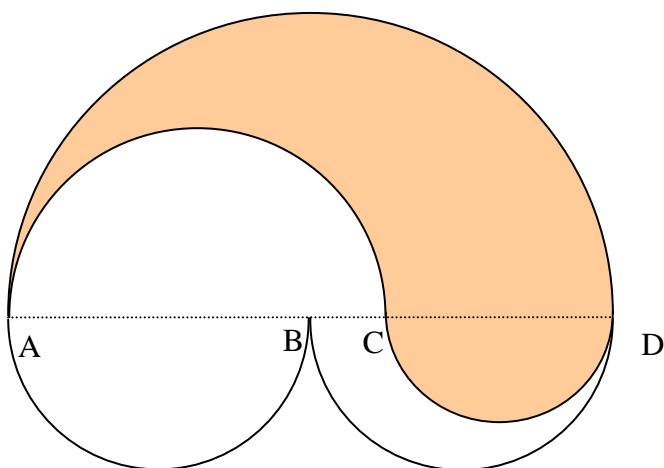
The figure shown is made up of 5 semicircles whose diameters all lie on the line joining A to D.
AB = BD = 8cm.
BC =2 cm.

Show that the shaded and unshaded shapes have

(a) the same perimeter;

(b) the same area.

You may leave your answers in terms of π .

7. a) Write the numbers 28, 44, 56 and 144 in their prime factorised form.

 b) Four cargo ships leave a port at the same time. The first ship returns every 28 days, the second every 44 days, the third every 56 days and the fourth every 144 days. Leaving your answer in prime factorised form, what is the least number of days before all four ships are again at the port together?

 c) A conference is to be attended by 672 students from Harvard, 504 from Yale and 2352 from Princeton. What is the greatest number of groups that can be formed so that the students from each of the three universities are distributed equally among the groups?

8. $\lfloor n \rfloor$ means the largest integer less than or equal to n , e.g. $\lfloor 2.3 \rfloor = 2$, $\lfloor 7 \rfloor = 7$ and $\lfloor 5.8 \rfloor = 5$.

 a) Evaluate the following:

   i) $\left\lfloor \dfrac{x}{4} \right\rfloor$ when $x = 15$ and also when $x = 16$;

   ii) $\left\lfloor \dfrac{5x}{3} \right\rfloor$ when $x = 14$ and also when $x = 15$.

 b) Solve $x + \dfrac{x}{4} = \dfrac{5x}{3} - 7$, giving your answer as a mixed number.

c) Find both solutions to $x + \left\lfloor \dfrac{x}{4} \right\rfloor = \left\lfloor \dfrac{5x}{3} \right\rfloor - 7$.

9. a) Expand and simplify $(x-y)^2$.

b) Using your answer to part (a), or otherwise, expand and simplify $(x-y)^2(x+y)$.

c) Given that $(x-y)^2 \geq 0$ for all values of x and y, use your answer to part (a) to show that

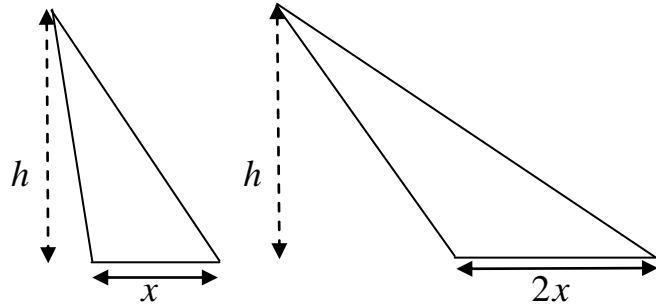$$\frac{x}{y} + \frac{y}{x} \geq 2,$$

for all positive numbers x and y.
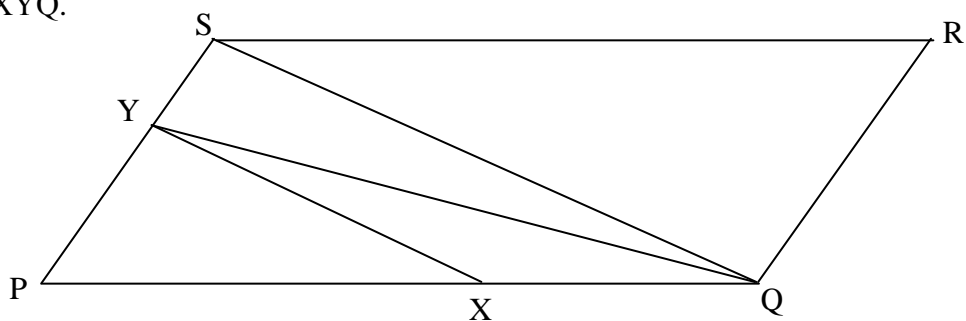
d) Using your answer to part (b), show that

$$\frac{x^2}{y} + \frac{y^2}{x} \geq x + y,$$

for all positive numbers x and y.

10. a) Calculate the area of each of the triangles opposite, leaving your answers in terms of x and h. h is the perpendicular height.

b) PQRS is a parallelogram of area q. X divides PQ in the ratio 2:1 and Y divides PS in the ratio 3:2. Calculate the areas of the following triangles in terms of q:
   i) ΔPSQ;
   ii) ΔPYQ;
   iii) ΔXYQ.

[End of paper]