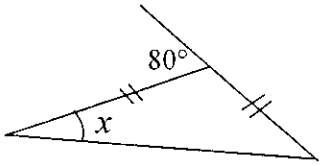
Eton College King's Scholarship Examination 2008

MATHEMATICS B **(One and a half hours)**

Answer as many questions as you can. Each of the ten questions carries ten marks. Show all your working. Calculators are not allowed.

1. a) Calculate the angle x in the isosceles triangle shown with an exterior angle of 80°.



b) The vertices of a regular nonagon are labelled A, B, C, D, E, F, G, H and I, in consecutive order. Calculate each of the angles in the triangle ACF.

2. a) Solve the simultaneous equations $198x - 202y = 3212$ and $202x - 198y = 3188$.

b) Hence solve the simultaneous equations

$$198p^2 + \frac{202}{q} = 3212 \quad \text{and} \quad 202p^2 + \frac{198}{q} = 3188.$$

3. a) A $4 \times 4 \times 4$ cube is made up of unit cubes. The outside of the cube is then painted by dipping it in paint. How many of the unit cubes will have exactly
i) 3 faces painted? ii) 2 faces painted? iii) 1 face painted? iv) 0 faces painted?

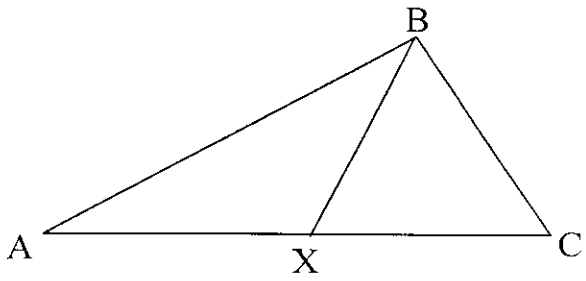
b) Repeat part (a) with an $x \times x \times x$ cube.

c) What would be each of your answers to part (b) if the eight unit corner cubes were removed and discarded and then the large *altered* cube was dipped?

4. This question refers to the diagram opposite.

a) If AX=BX=BC and angle BAX = x, prove angle BCX = $2x$.

b) If instead AX=BX=CX, prove that angle ABC is a right angle.



5. a) Solve the equation $\frac{1}{3}x = x - (5 - \frac{1}{6}x)$.

b) A ship is twice as old as its boiler was when the ship was the age the boiler is now. The present combined age of the ship and boiler is 49 years. How old is the ship now?

6. a) A car travels from Eton to Ascot with an average speed of 50 km/hour on the outward journey and with an average speed of 75 km/hour on the return journey. Show that the average speed for the whole journey is 60 km/hour.

b) Normally, the train between Windsor and Feltham travells at an average speed of 90 km/hour. If it is delayed and arrives 4 minutes late, its average speed drops to 70 km/hour. What is the distance between the stations of Windsor and Feltham?

7. Two common Pythagorean triples are (3, 4, 5) and (5, 12, 13) because $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. We will now find other whole number solutions of Pythagoras' theorem, $x^2 + y^2 = z^2$, in the special case that $z = y + 1$.

a) Substitute $z = y + 1$ into Pythagoras' theorem, to eliminate z. Rearranging your new equation, show that $y = \frac{1}{2}(x^2 - 1)$.

b) Using this formula, explain why x must be an *odd* whole number.

c) Hence calculate the first seven Pythagorean triples, of this type, starting with $x = 3$.

8. We know that $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ and $\frac{1}{12} + \frac{1}{12} = \frac{1}{6}$.

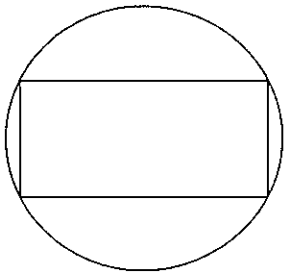
We wish to solve $\frac{1}{m} + \frac{1}{n} = \frac{1}{6}$, for m and n whole numbers with $m \geq n$.

a) Show that the above equation can be rearranged into the form $(m-6)(n-6) = 36$.

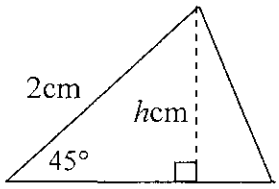
b) Hence solve for all whole number pairs of values m and n with $m \geq n$.

9. a) Suppose $xy = 16$ and $x + y = 11$, evaluate $x^2 + y^2$.

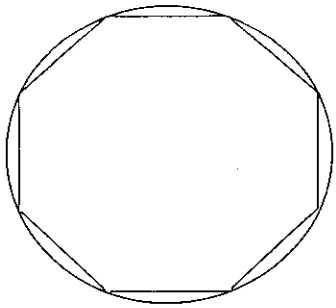
b) A rectangle is inscribed in a circle of radius 7cm. If the perimeter of the rectangle is 36cm, calculate the area of the rectangle.



10. a) Calculate the exact perpendicular height, h, of the isosceles triangle shown



b) A regular octagon is inscribed inside a circle of radius 2cm. Calculate the area of the octagon.



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