Eton College King's Scholarship Examination 2008

(One and a half hours)

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all of your working. Calculators are allowed on this paper.

1. **Compulsory Question**

(a) Find the positive value of v where $v^2 = u^2 + 2as$ and $u = 5$, $a = 3$ and $s = 4$. [2]

(b) Solve the following inequalities :

(i) $\frac{x+3}{2} > 11$

(ii) $17 - 3x > 29$ [2, 2]

(c) Calculate the following:

(i) 25% of £36

(ii) 17.5% of £3000 [1, 1]

(d) Solve the following:

(i) $\frac{1}{2}(x+5) = 3$

(ii) $\frac{3}{2}(x-1) + \frac{1}{5}(x+2) = 4$ [1, 2]

(e) (i) Find the 50th term of the sequence 3, 5, 7, 9…

(ii) Which term of the sequence 3, 5, 7, 9… is equal to 249? [1, 2]

(f) James decides that he wants to make a pudding.

(i) If his recipe book tells him that he requires 40g of sugar for 6 people then how much sugar does James need for 9 people? [2]

(ii) James buys the sugar from the supermarket, which was offering the sugar at a reduced price of 64p. If its original price was 80p then find the percentage discount for the sugar. [2]

(iii) If the 40g of sugar needed for 6 people was 8% of the bag of sugar then how much did the bag hold? [2]

Please turn over

(g) A triangle has angles of x, $2x-10$ and $3x+40$. Find the value of x. [3]

(h) (i) A cuboid measures 5cm by 6cm by 10cm. A cylinder has radius 5cm and height 4cm. Which has the larger volume? [3]

(ii) A 15cm by 20cm rectangle forms the base of a box of height 8cm. If the box is filled up with water and the water then poured into containers measuring 3cm by 4cm by 2cm then how many such containers will be filled? [2]

(i) Solve the following simultaneous equations:

$$5x-3y=26$$
$$3x+2y=27$$

[4]

(j) A set of 19 boys did a test (marked out of 50) in which the mean mark was 35. The 20th boy in the set then did the test.

(i) What is the largest possible value of the mean mark for all 20 boys? [2]

(ii) Given that the mean mark for all 20 boys decreased to 34, find the score that the 20th boy got in the test. [2]

(k) Two dice, one red and one blue, are rolled. Find the probability that:

(i) The red dice shows a 5. [1]

(ii) The sum of the scores is 12. [2]

(iii) The two dice show the same score. [2]

(l) A rectangular field is three times longer than it is wide. If it is 40m wide then find the greatest distance (to 3sf) between any two points in the field. [2]

(m) Calculate the following, <u>showing all your working</u>:

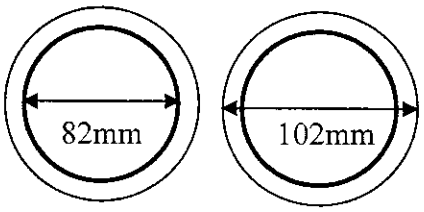
(i) $3\frac{2}{5}+4\frac{1}{3}$

(ii) $2\frac{1}{4}+3\frac{1}{9}$ [2, 2]

(n) Evaluate the following:

$\left(1+\frac{1}{2}\right)\times\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{4}\right)\times\left(1+\frac{1}{5}\right)\times....\times\left(1+\frac{1}{19}\right)$ [3]

2. A stationery shop sells a large roll of adhesive tape. The roll has 66m of tape on it. A boy is asked to find the thickness of the tape.
He uses a ruler to find that the cardboard disc around which the tape is wound has diameter 82mm and that the overall diameter (with the tape) is 102mm.



(a) Find (to 3sf) the cross sectional area (shaded above) of the tape (in mm^2). [2]

(b) Hence find (to 3sf) the thickness of the tape. [4]

The boy now wants to find the thickness of the tape when the roll has length lm, the diameter of the core is $d$mm and the overall diameter (with the tape) is Dmm.

(c) Find an expression for the thickness, T (in mm), of the tape. [4]

3. (a) Expand and simplify the following:

(i) $(x+8)(x-8)$

(ii) $\dfrac{(x^2+2x)}{x}$

(iii) $(x-1)(x^2+x+1)$

[1, 2, 2]

(b) Without using a calculator and without any complicated calculations, use (a) to evaluate the following (showing all your working clearly):

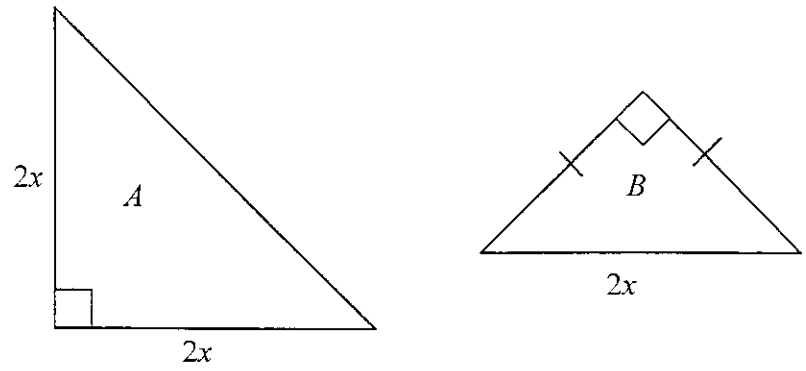
(i) 2008×1992

(ii) $(2008^2+4016)\div2008$

(iii) $1999\times(2000^2+2001)$

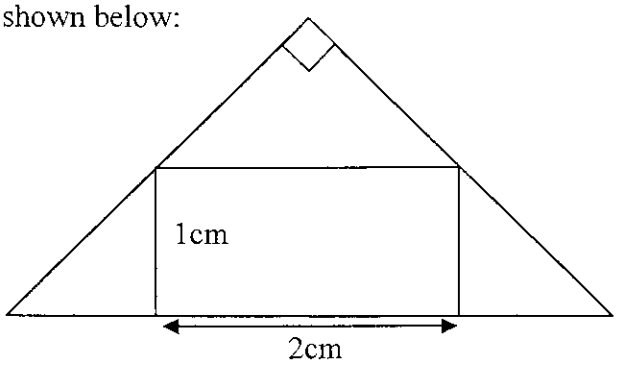
[2, 1, 2]

4. (a) Below are two right-angled isosceles triangles A and B:



Find the area of the two triangles A and B in terms of x. [2, 3]

(b) A rectangle measuring 1cm by 2cm is put inside a right-angled isosceles triangle so that its 2cm base lies on the base of the triangle and so two of its corners touch the triangle, as shown below:



Find the area of the large triangle. [5]

5. (a) Expand and simplify $\left(x+\frac{1}{x}\right)^2$ [2]

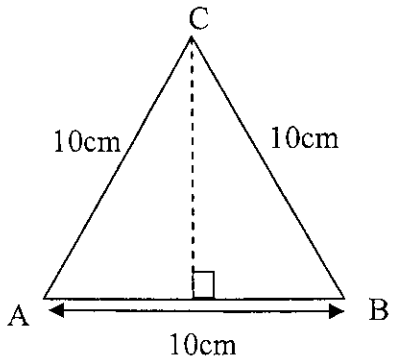
(b) x is now chosen in such a way that $x+\frac{1}{x}=4$

(i) Find the value of $x^2+\frac{1}{x^2}$ [2]

(ii) Simplify $\left(x^2+\frac{1}{x^2}\right)^2$ to find the value of $x^4+\frac{1}{x^4}$ [3]

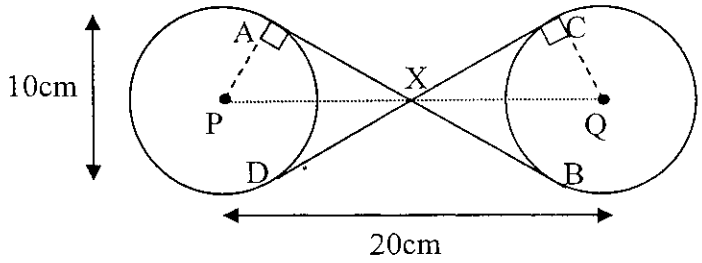
(iii) Find the value of $x^{16}+\frac{1}{x^{16}}$ [3]

6.



(a) Show that the height of the equilateral triangle ABC shown above is $5\sqrt{3}$ cm. [2]

The diagram below shows two circular discs of diameter 10cm. The points P and Q are the centres of the two discs. P and Q are 20cm apart. A taut belt passes over both discs as shown below



The points A, B, C and D above are the points at which the contact between the belt and the disc is lost.

In the following, assume that the belt is infinitely thin and so ignore any problem of the belt crossing over itself at X.

(b) Write down the lengths AP and PX. [1]

(c) Use (a) to write down the values of the angle XPA and the length AX. [2]

(d) Calculate the total length of the belt (leaving your answer in the form $a\sqrt{b}+\frac{p}{q}\pi$ where a, b, p and q are all positive whole numbers). [5]

END OF PAPER