Eton College King's Scholarship Examination 2007

MATHEMATICS B **(One and a half hours)**

Answer as many questions as you can. Each of the ten questions carries ten marks. Show all your working. Calculators are not allowed.

1. (a) What is the value of $\sqrt{-((1\times2\div(3\times4)-5)\times6-7)}\div(8\times9)$?

   (b) When a barrel is 30% empty it contains 30 litres more than when it is 30% full. How many litres does the barrel hold when full?

   (c) Split the number 68 into two parts such that $\frac{4}{7}$ of one part is equal to $\frac{2}{5}$ of the other.

2. (a) If eight lorries can transport 450 tonnes of gravel in 12 hours, how long does it take six similar lorries to transport 720 tonnes of gravel, working at the same rate? Give your answer in hours and minutes.

   (b) Three barrels contain mixtures of wine and water in the ratio 1:2, 3:2 and 2:5 respectively. A new mixture is made by scooping a proportion from each barrel in the ratio 3:5:4 respectively. What is the ratio of wine to water in the new mixture?

3. A teacher writes a positive whole number less than 4000 on the blackboard. One boy states that the number is a multiple of 2; a second that it is a multiple of 3; and so on consecutively until the eleventh boy says that it is a multiple of 12. The teacher remarks that all except two of the boys were right and, moreover, that the two who were wrong spoke one after the other. What was the number that the teacher wrote on the blackboard? You must explain your reasoning carefully in this question.

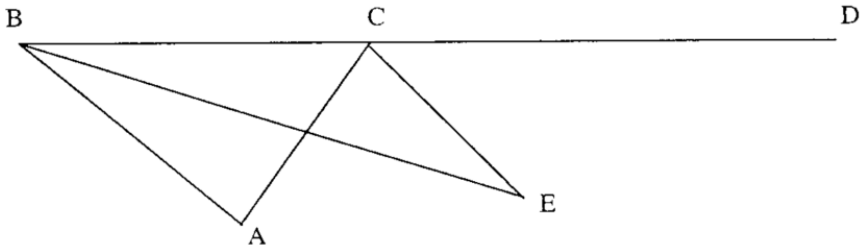
4. Solve the simultaneous equations:

   (a) $\frac{2}{x}-\frac{3}{y}=7, \quad \frac{8}{x}+\frac{9}{y}=91$

   (b) $2^{p+1}-3^{2q+1}=7, \quad 2^{p+3}+3^{2q+2}=91$

5. (a) From coastguard station F a ship is seen on a bearing of 055°. As seen from the ship, the angle between the directions of coastguard station F and coastguard station G is 140°. What are the possible bearings of the ship from coastguard station G?

   (b) In the diagram below, BCD is a straight line, BE bisects angle ABC and CE bisects angle ACD. Prove that angle BAC is twice angle BEC.

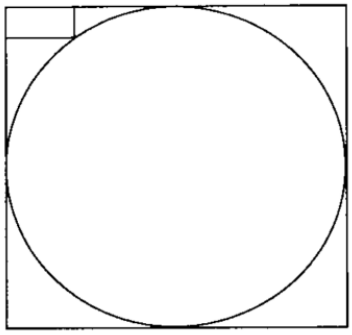


6. The diagram shows a circle, of radius r, inscribed inside a square and a 1cm by 2cm rectangle inscribed in the top left corner between the circle and the square.

(a) Show that r satisfies the equation $r^2 - kr + 5 = 0$, where k is a constant to be found.

(b) Hence, by completing the factorisation $(r-1)(\ldots - \ldots) = 0$, calculate the radius of the circle.

(c) Find a similar equation, in the case where the rectangle is 1cm by 3cm, and verify that $r = 4 + \sqrt{6}$ satisfies this equation.



7. (a) Prove that the difference between a number '*ab*' and its reverse '*ba*' is never prime.

(b) A *palindromic number* is one that reads the same when its digits are reversed, such as 5115. What is the largest six-digit palindromic number that is exactly divisible by 6?

8. Only two rectangles have dimensions that are integers and their area and perimeter are numerically equal. Let x be the length and y the width of the rectangles.

(a) Show that x and y satisfy $(x-2)(y-2) = 4$ and hence find the dimensions of the rectangles.

Using a similar approach we now wish to find all the rectangles whose dimensions are integers, and whose area is numerically equal to three times its perimeter.

(b) Show that x and y now satisfy $(x-k)(y-k) = k^2$, where k is a positive integer to be found.

(c) Hence find the dimensions of all the rectangles that have their area numerically equal to three times their perimeter.

9. When Roald Dahl had finished his first book, he noticed that the number of digits he used to number the pages (starting from page 1) was an exact multiple of the number of pages in the book. If the book contains over 100 pages but fewer than 1000:

(a) Show that 192 digits are used to number the pages between 1 and 100 inclusive.

(b) If there are x pages, show that the number of digits used to number the pages is $3x - 108$.

(c) Hence find the number of pages in the book and the total number of digits used to number the pages.

10. (a) Using standard British coins (1p, 2p, 5p, 10p), it is possible to pay a total of 10p in many ways; for example ten 1p coins. In how many different ways can one pay 10p?

(b) Hence or otherwise, in how many different ways can one pay 20p using standard British coins (1p, 2p, 5p, 10p, 20p)?

(End of paper)