Eton College King's Scholarship Examination 2007

MATHEMATICS A **(One and a half hours)**

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all of your working.

1. **Compulsory Question**

(a) Solve the following simultaneous equations:

$$4x+3y=29$$
$$5x+2y=31$$ [4]

(b) Two houses increased in value by 5% over 2006.

(i) One of the houses was worth £150,000 on 1st January 2006.
How much was it worth on 1st January 2007? [1]

(ii) The other house was worth £420,000 on 1st January 2007.
How much was it worth on 1st January 2006? [2]

(c) Solve the following inequality : $19-3x<34$. [3]

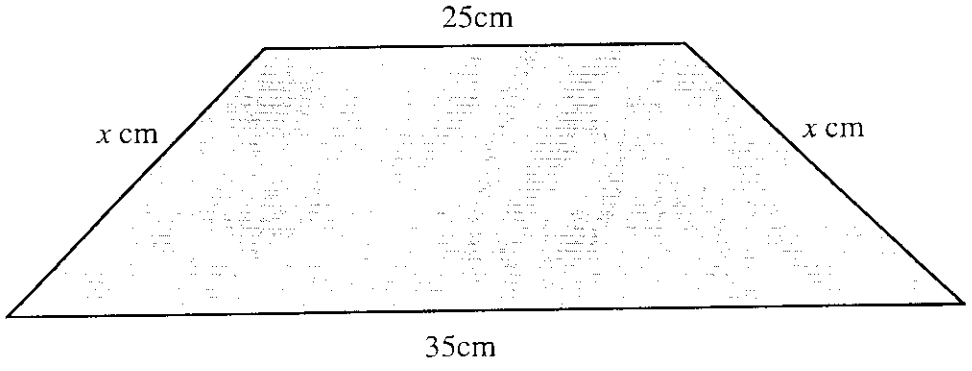
(d) Find the value of s where $s=ut+\frac{1}{2}at^2$ where $u=5$, $a=4$ and $t=3$. [2]

(e) A car's petrol consumption is 42 miles per gallon.
One gallon is 4.546 litres, one mile is 1.6km.

(i) What is the car's petrol consumption in km per litre? [3]

(ii) If petrol costs 90.9 p per litre then how much would the petrol cost for a journey of 800km? [4]

(f)



(i) Given that the area of the above trapezium is 300 cm^2, find its height. [2]

(ii) Find the value of x. [2]

(g) Simplify the following as far as possible:

(i) $(x+2)(x-2)+x(x-4)+4(x+1)$ [2]

(ii) $(x-2)(x+3)-(x+2)(x-3)$ [3]

Turn over for the remainder of question 1

(h) Each exterior angle of an n-sided regular polygon is $\frac{D}{n}$ where D is a number that does not change for different values of n.

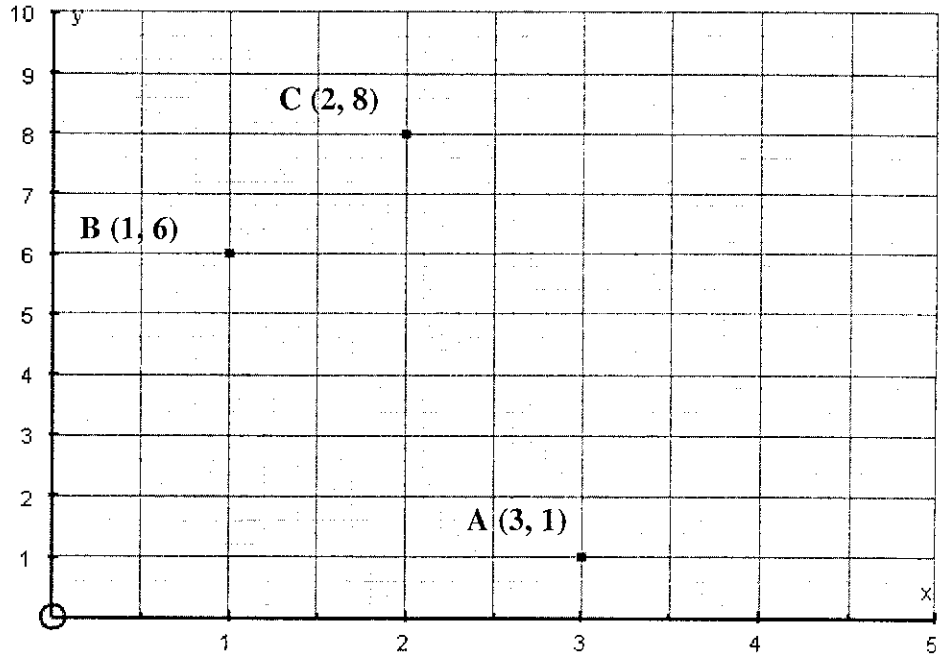
(i) What is the value of each exterior angle of an equilateral triangle? [1]

(ii) Use this to find the value of D. [1]

(iii) Hence find the value of each exterior angle of a regular hexagon. [1]

(iv) Each exterior angle of a regular polygon is 18°. How many sides does it have? [1]

(i) (i) Find the distance between the points A and B shown below. [3]



(ii) Find the coordinates of the point D such that ABCD (labelled in that order) is a parallelogram. [2]

(j) (i) Find the mean of the following numbers : 5, 7, 10, 6, 12. [2]

(ii) If 12 was replaced with 52 then what would happen to the median of the numbers? [1]

(k) Solve the following:

(i) $\frac{1}{3}x+5=7$

(ii) $\frac{2}{5}(3x+1)=4$

(iii) $\frac{2}{3}(x+1)+\frac{1}{4}(x-1)=3$

[2, 2, 3]

(l) Charlie got some sweets for his birthday. He gives $\frac{1}{3}$ of his sweets to John. John then gives $\frac{4}{5}$ of these sweets to Harry. If John was left with 4 then:

(i) How many did Charlie have left? [2]

(ii) How many was John given? [1]

2. On the back of every modern book you can find an ISBN code. This is a ten digit number which uniquely defines the book such as the one below.

On the back of Tolkein's *The Return of the King* is the ISBN code 0007203608.

Take the first digit of the ISBN code and multiply it by 1, the second digit and multiply by 2, the third digit and multiply by 3. When you carry this on and add them all together you get what we will call the book's ISBN check number. We call it the ISBN check number because a ten digit number is a valid ISBN number if and only if this check number is a multiple of k, where k is a whole number that is the same for all books.

(a) Show that *The Return of the King*'s ISBN check number is 187. [2]

(b) What is the ISBN check number for Tolkein's *The Two Towers*, whose ISBN code is 0007203594? [2]

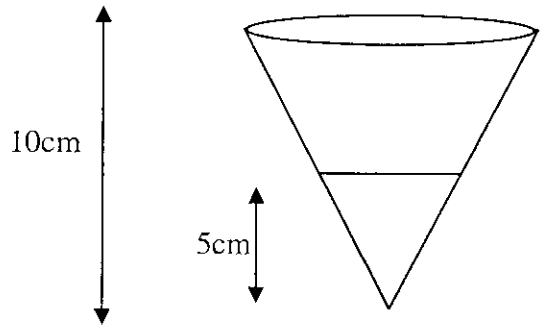
(c) Given that all ISBN check numbers are multiples of the whole number k, use (a) and (b) to find the value of k. [2]

(d) (i) A boy rings up a bookshop to order *The Fellowship of the Ring*. He reads the ISBN number out as 0007203581. Explain why the bookshop's computer will state that this ISBN is not valid. [1]

(ii) The boy realises that the last digit of the ISBN was wrong. What should it have been? [3]

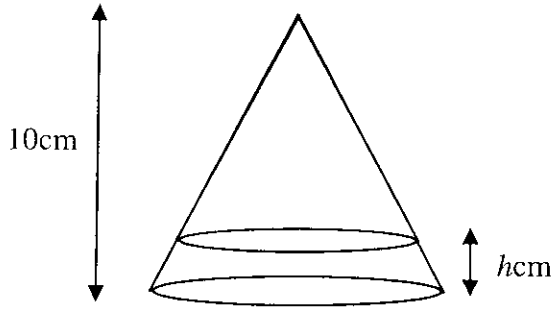
3. (a) If the radius and the height of a cone are doubled in length then by what factor will the volume of the cone be increased? [3]

(b) 100 cm^3 of water is poured into the cone below. It comes to a height of 5cm, which is half the height of the cone.



(i) How much water would be in the cone if it was filled to the top? [2]

(ii) There is a lid on the cone and it is now turned over.



What will the height of the water be in the cone? (Give your answer to 3 significant figures.) [5]

4. (a) Simplify the expression $(a-b)^2+(a-c)^2+(b-c)^2$ as far as possible. [2]

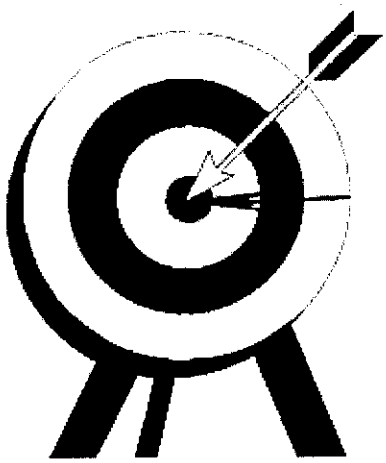
(b) Hence prove that $a^2+b^2+c^2 \geq ab+ac+bc$. [2]

(c) Show that this inequality holds for $a=5$, $b=3$ and $c=1$. [1]

(d) Multiply both sides of the inequality in (b) by $a+b+c$, where $a+b+c \geq 0$. Use this to copy and complete the following inequality, expressing the right hand side as simply as possible:

$a^3+b^3+c^3 \geq .....$ (where a, b and c are all positive numbers). [5]

5. In an archery competition decider, Arthur shoots an arrow and Brian shoots an arrow. If one of them hits the bullseye and the other one doesn't then the one who hit the bullseye is the winner. Otherwise they fire an arrow each again. This process continues until one hits and the other misses.
The probability of Arthur hitting the bullseye with his first arrow is 0.45. The probability of Brian hitting the bullseye with his first arrow is 0.4.



(a) Show that the probability that neither Arthur nor Brian hits with their first arrow is 0.33. [1]

(b) Find the probability that Arthur wins the competition by only firing one arrow. [2]

(c) Find the probability that Brian wins the competition by only firing one arrow. [2]

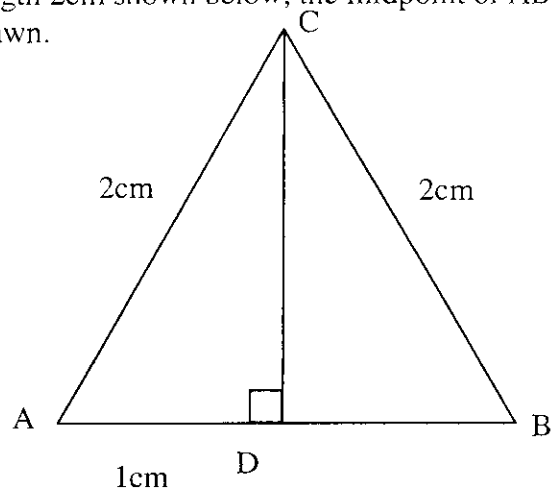
If Arthur misses the bullseye with one arrow then the probability of his hitting the bullseye with his next arrow falls to a value of 0.2.
If Brian misses the bullseye with one arrow then the probability of his hitting the bullseye with his next arrow falls slightly to a value of p.

(d) What is the probability, in terms of p, that Brian wins with his second arrow? (i.e. Arthur and Brian both miss with their first arrow, Arthur misses with second arrow and then Brian hits with second arrow). [2]

(e) Find the value of p such that the probability of a competitor winning with either his first or second arrow is the same for both Brian and Arthur. [3]

6. **All answers to this question should be given as exact values.**
For example, if the answer is $\sqrt{2}$, then this should be given as $\sqrt{2}$ and not as 1.41 (to 3 significant figures).

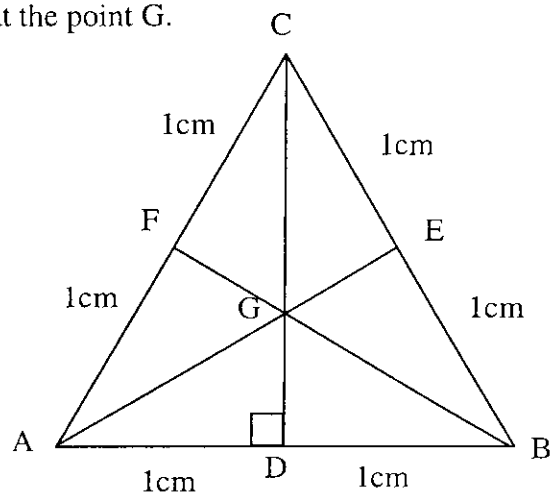
In the equilateral triangle ABC of side length 2cm shown below, the midpoint of AB is marked as D and the line CD has been drawn.



(a) Find the *exact* length of CD. [2]

(b) Hence find the *exact* area of the triangle ABC [2]

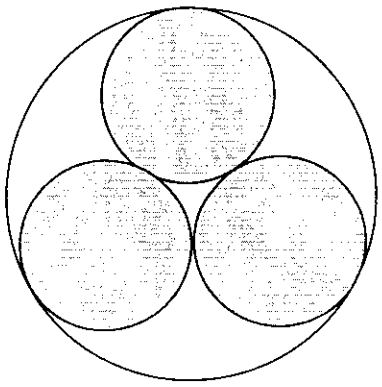
In the diagram below the midpoint of BC is marked as E and the midpoint of AC is marked as F. AE, BF and DC all meet at the point G.



(c) By considering the area of the triangle AGB, find the exact length of GD. [2]

In the diagram below the three smaller circles all have radius 1cm.

(d) What is the *exact* value of the radius of the larger circle? [4]



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