Eton College King's Scholarship Examination 2006

MATHEMATICS B **(One and a half hours)**

Answer as many questions as you can. Each of the ten questions carries ten marks. Show all your working. Calculators are not allowed.

1. (a) Put the following numbers in ascending order:

$$\sqrt{9} \quad \pi \quad \sqrt{\frac{49}{4}} \quad 2\sqrt{2} \quad \frac{17}{5}$$

(b) Express the following as fractions in their simplest form:

$a = 0.\dot{3}$

$b = 0.025$

$c = 0.\dot{2}\dot{7}$

2. (a) Find the value of $\sqrt{1+2+3+4+x}$ when $x = 15$.

(b) Find the whole number values of x (which may be either positive or negative) such that $x < 15$ and $\sqrt{1+2+3+4+x}$ is a positive whole number.

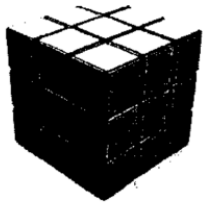
3. (a) What is the sum of the internal angles of a 3 sided polygon (i.e. a triangle)?

(b) What is the sum of the internal angles of a 4 sided polygon (e.g. a rectangle)?

(c) What is the sum of the internal angles of an n sided polygon?

(d) Two internal angles of a polygon are $50°$ and $150°$. All its other internal angles are $160°$. How many sides does the polygon have?

4. The diagram below shows a $3\times3\times3$ cube that is made up of a set of small cubes, each measuring $1\times1\times1$.



Now consider a large cube measuring $n\times n\times n$ that is made up of a set of small cubes, each measuring $1\times1\times1$. The outside of the large cube is painted and then dismantled back into the $1\times1\times1$ small cubes.

How many of the $1\times1\times1$ cubes will have:

(a) exactly three sides painted;

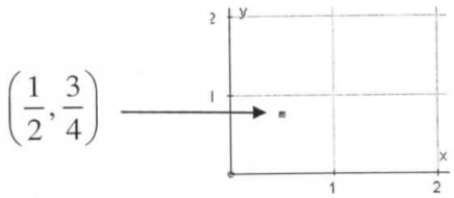
(b) exactly two sides painted (leave your answers in terms of n);

(c) no sides painted (leave your answers in terms of n)?

[Please turn over]

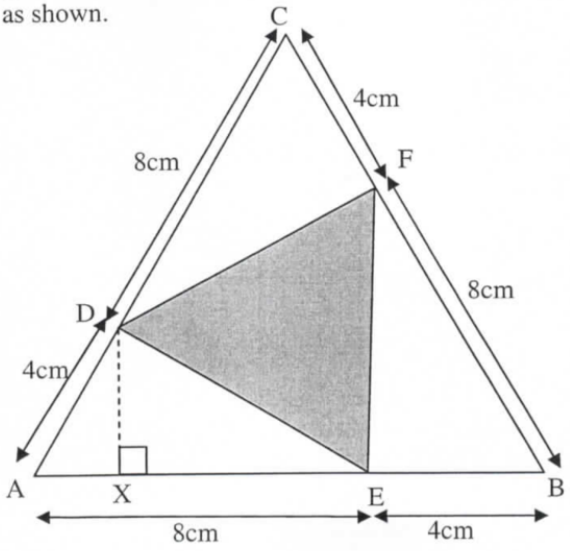
5. Adam and Brian are two runners. Adam is faster than Brian. Adam runs 1 mile in a minutes and Brian runs 1 mile in b minutes.
   (a) How far does Brian run in a minutes?
   (b) If they both run for five minutes, then how much further has Adam run than Brian?
   (c) If $b = 10$ and over forty minutes Adam runs a mile further than Brian, then find a.

6. x and y are positive numbers chosen at random that lie between 0 and 2.
   If, for example, $x = \frac{1}{2}$ and $y = \frac{3}{4}$ then this could be represented on the diagram shown below by the point $\left(\frac{1}{2}, \frac{3}{4}\right)$.



   (a) Draw a sketch of the above diagram and draw on it the two lines that represent all the pairs of values of x and y in which x and y differ by exactly 1.
   (b) Hence calculate the probability that x and y differ by less than 1.

7. In the diagram shown below the triangle DEF has all its vertices lying on the sides of the equilateral triangle ABC with side length 12cm. D, E and F are all one third of the way along the sides as shown.



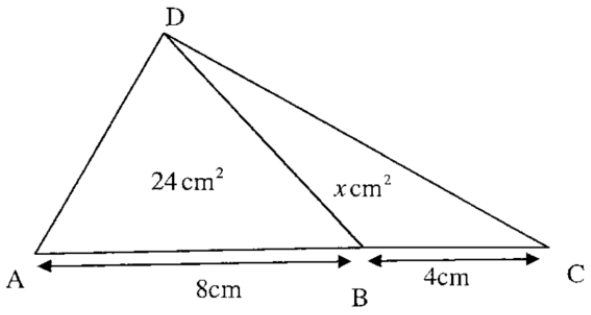
   (a) By considering the triangle AXD as half an equilateral triangle, write down the length of AX.

(b) Use your answer to (a) to show that the length of DX is $2\sqrt{3}$ cm.

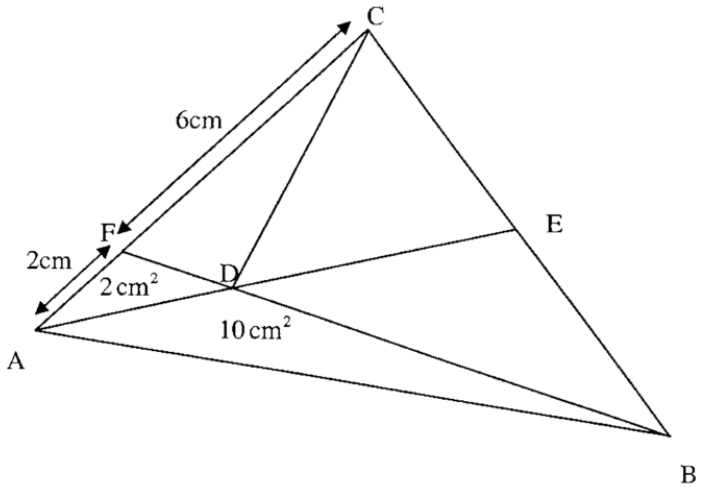
(c) Use (a) and (b) to find what proportion of the triangle ABC lies in the triangle DEF (by first calculating the length of DE).

8.

(a) In the diagram shown the triangle ABD has area $24\,\text{cm}^2$. Use the fact that the triangles ABD and BCD have the same height to find x, the area of the triangle BCD.



(b) In the diagram shown below the triangle ABC is split up into different shapes. The triangle ADF has area $2\,\text{cm}^2$, the triangle ABD has area $10\,\text{cm}^2$. ADE and BDF are straight lines. Find the area of the triangle DBC.



[Please turn over]

9. (a) Factorise $m^2 - s^2$.

(b) If $m^2 - s^2 = 57$ and $m > s$ then find two possible pairs of positive whole number values for m and s.

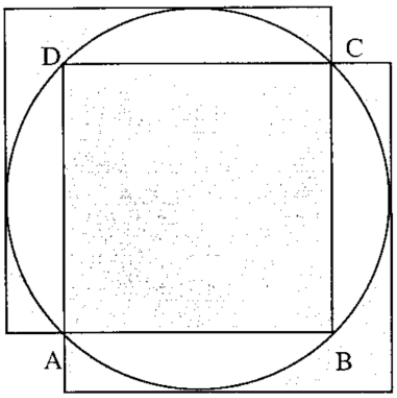
A company sells a variety of square jigsaw puzzles which are made up of square pieces. They sell the puzzles in three sizes: small, medium and large. The medium sized puzzle has 57 more pieces than the small puzzle and the large puzzle has 203 more pieces than the medium puzzle. A 25×25 square is considered the largest possible square for these sorts of puzzles.

If the small puzzle has s^2 pieces, the medium puzzle has m^2 pieces and the large puzzle has l^2 pieces.

(c) Explain why $m^2 - s^2 = 57$ and write down an equation involving l and m.

(d) Solve the equation involving l and m to find the values of s, l and m.

10. In the diagram shown below the circle has radius 4cm and the square ABCD is the largest square that can be drawn inside the circle. A, B, C and D all lie on the circle. The two other squares are such that their sides touch the circle as shown.



(a) Find the area of the smaller square.

(b) Find the area of one of the larger squares, leaving $\sqrt{2}$ in your answer.

(c) Find the shaded area, leaving π and $\sqrt{2}$ in your answer.

END OF PAPER