# Eton College King's Scholarship Examination 2006

**MATHEMATICS A** **(One and a half hours)**

*Answer Question 1 and as many of the other five questions as you can.*
*Question 1 is worth 50 marks. All other questions are worth 10 marks each.*
*Show all of your working.*

1. **Compulsory Question**

(a) (i) Increase £66 by 15%. [2]

(ii) Calculate the original price of an item selling at £79.20 in a "10% off" sale. [2]

(b) Simplify fully

(i) $(2x+1)(x-2)+2$, [2]

(ii) $\dfrac{8y^2-4xy^3}{2y}$, [2]

(iii) $2r-3(s-2r-(r-s))$. [2]

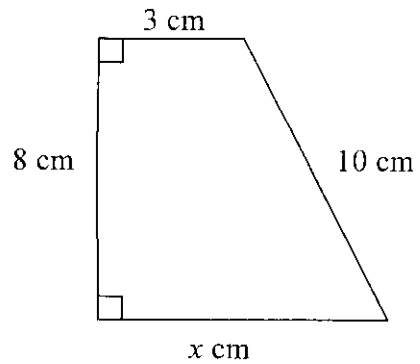
(c) (i) If I divided £810 in the ratio 2:3:4, what is the value of the smallest share? [2]

(ii) If $\frac{3}{5}$ of the pupils at a school are boys, what is the ratio of boys to girls? [2]

(d) Solve the pair of simultaneous equations

$$3x-2y=12$$
$$2x+5y=-11.$$ [4]

(e) Considering the shape below,




(i) calculate the value of $x$, [2]

(ii) calculate the area of the shape. [2]



(Question 1 continued on next page)

(f) (i) Make $m$ the subject of the formula $y = 2mx + c$. [2]

(ii) Make $x$ the subject of the formula $\dfrac{x-y}{2} = ax + b$. [3]

(iii) Make $g$ the subject of the formula $b = 2a\sqrt{\dfrac{g}{m}}$. [3]

(g) Factorise fully

(i) $10x^2yz^3 + 2xyz^2$, [2]

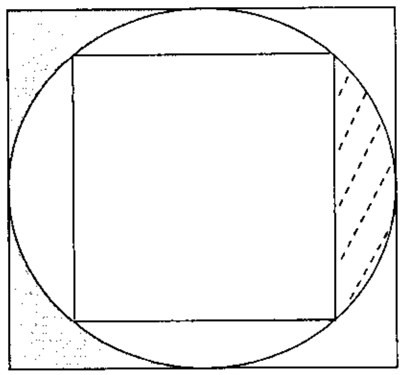
(ii) $\dfrac{6x^2}{5y} - \dfrac{3x}{10y^2}$. [2]

(h) Solve, leaving your answers in fractional form

(i) $6\left(3x + \frac{2}{3}\right) = \frac{2}{5}(x+1)$, [3]

(ii) $13 + x - \frac{2}{3} \geq -2x$. [3]

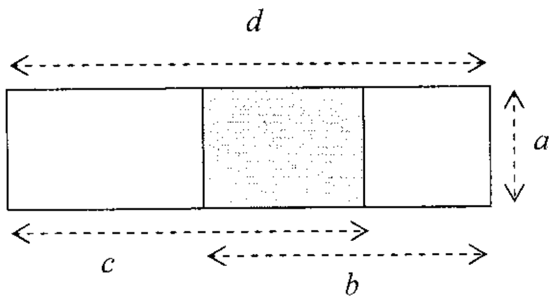
(i) In the diagram below, *which is NOT drawn to scale*, the circle has a radius of 4 cm. It is inscribed inside a square and also has a square inscribed inside it as shown below.



(i) Show that the area of the <u>heavily shaded</u> region is $6.87\,\text{cm}^2$ (to 2d.p.). [3]

(ii) Calculate the area of the region marked with the <u>dotted lines</u> (to 2d.p.). [3]

(j)




(i) Write down an expression, in terms of $a$, $b$, $c$ and $d$, for the area of the shaded region. [3]

(ii) Calculate this area, when $a$=2, $b$=5, $c$=3, $d$=6. [1]

2. A reservoir is to be made so that it will hold $3.2 \times 10^7 \text{m}^3$ of water when full. A 1:200 scale model is built in order to make certain calculations.

(a) When the model is full, the greatest depth is 18cm. What will be the greatest depth, in metres, of the reservoir? [2]

(b) If the surface area of the water in the model is $300\text{cm}^2$, calculate the corresponding surface area of the water in the reservoir, giving your answer in $\text{m}^2$. [3]

(c) What will be the volume, in $\text{m}^3$, of water in the model when full? [2]

(d) The volume of the reservoir was not given precisely at the start of the question. It was given to the nearest thousand cubic metres. Specially treated water is to be used to fill the reservoir, and it has a mass of $3.58 \times 10^7$ tonnes (this figure only being correct to 2 decimal places). Calculate the <u>maximum</u> density of the treated water, giving your answer in $\text{kg/m}^3$ and to 2 decimal places. [3]
[Density = Mass / Volume, 1 tonne=1000 kg]

3. A 'perfect square' has the property that every row, column and diagonal add up to the same number, $S$.

(a) Consider the square below.

| | | |
|---|---|---|
| 8 | $a$ | $b$ |
| $c$ | 6 | 5 |
| $d$ | $e$ | $f$ |

By solving the equations

$$c+6+5=8+c+d$$
$$8+6+f=b+5+f$$
$$d+6+b=S$$

or otherwise,

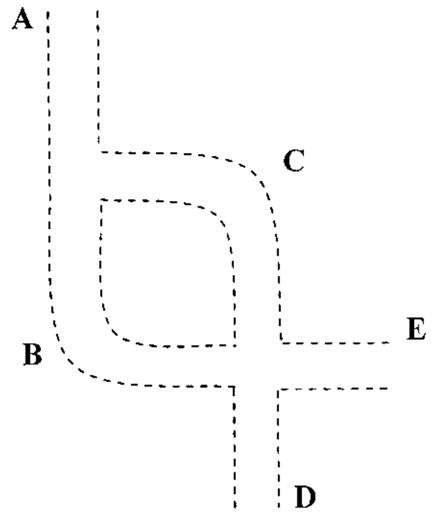
(i) find the values of $b$, $d$, and $S$ that make the square 'perfect', [3]
(ii) complete the square. [2]

(b) Find the values of $p$, $q$, $r$, $s$, $t$, and $u$ that make the square below 'perfect'.

| | | |
|---|---|---|
| $p$ | $q$ | $r$ |
| 3 | 7 | $s$ |
| 12 | $t$ | $u$ |

[5]

4. A rat is in the maze represented in the diagram below.




The rat wishes to go from point **A** to point **D**. When faced with a junction the rat is *three* times as likely to continue straight on as it is to turn. Where both left and right turns are possible they are *equally* likely. He never turns back.

(a) Write down the following probabilities,

(i) that he will go from A to B, [1]

(ii) that, if he goes via B, he will turn towards D. [1]

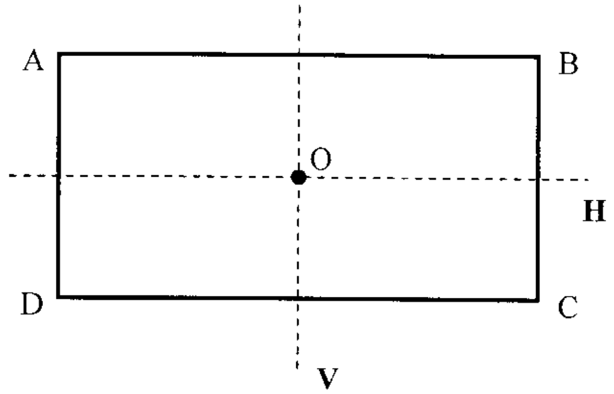
(b) Calculate the following probabilities of his reaching his destination, D,

(i) by the route ABD, [2]

(ii) by either of the *most obvious* routes, [3]

(iii) by the route ABCBD assuming that he *does* learn by his mistakes during that journey. [3]

5. Consider the rectangle ABCD drawn below.




Also drawn are horizontal and vertical mirror lines, **H** and **V** respectively, and the centre of the rectangle, O.
Two rotations of the rectangle are denoted by $R_{360}, R_{180}$ and represent rotations of 360 and 180 degrees respectively, centre O, in a clockwise direction. Two reflections of the rectangle are denoted by $M_H, M_V$ and represent reflections of the rectangle in the horizontal and vertical mirror lines respectively.
We can combine the transformations. For example, $R_{180}$ followed by $M_H$ is equivalent to $M_V$ (this can be seen by studying the positions of letters A,B,C and D).
This information has been recorded in the table below.

(a) Copy and complete the table below. [3]

| | | **Applied Second** | | | |
|---|---|---|---|---|---|
| | | $R_{360}$ | $R_{180}$ | $M_H$ | $M_V$ |
| | $R_{360}$ | ? | ? | ? | ? |
| **Applied First** | $R_{180}$ | ? | ? | $M_V$ | ? |
| | $M_H$ | ? | ? | ? | ? |
| | $M_V$ | ? | ? | ? | ? |

(b) An *identity* transformation is one which leaves the shape unchanged. State the identity for this set of transformations of the rectangle. [1]

(c) An *inverse* of a particular transformation is one which undoes what the transformation did (in other words combines with the transformation to produce the identity).
What do you notice about the inverses of each of these transformations? [1]



(Question 5 continued on next page)

We shall use the symbol $\times_5$ to stand for the operation called *multiplication modulo 5* which is defined by multiplying and then selecting the remainder upon division by 5.

For example, 2 combined with 3 is written as $2\times_5 3$ and equals 1 since $2\times3=6$ which leaves a remainder of 1 when divided by 5.

(d) Copy and complete the table for $a\times_5 b$ below for numbers 1,2,3,4, which has the above example entered already in bold.

| | | ***b* values** | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| | 1 | ? | ? | ? | ? |
| ***a* values** | 2 | ? | ? | **1** | ? |
| | 3 | ? | ? | ? | ? |
| | 4 | ? | ? | ? | ? |

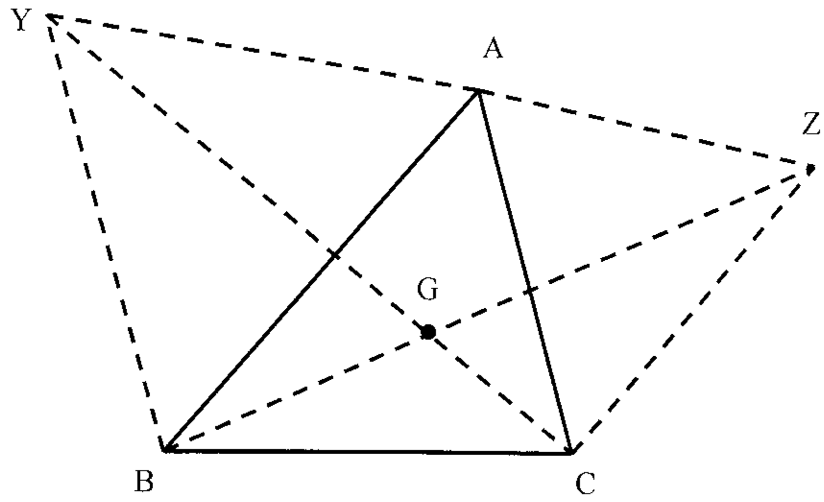
[3]

(e) One final table of an unknown set of transformations is given below, where $e$ represents the *identity*.

| | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $a$ | $e$ |
| $c$ | $c$ | $b$ | $e$ | $a$ |

State <u>with a reason</u> which of the two tables in either part (a) or (d) this table can be considered to be equivalent.
(Hint: you should consider inverses of elements in each table). [2]

6. Consider the diagram shown below.




Initially, the triangle ABC has equilateral triangles AYB and AZC drawn on the sides AB and AC respectively. The intersection of the lines BZ and CY is labelled G.
The rotation that maps Z onto C, centre A, is denoted by $\mathbb{R}$ .

(a) (i) Explain why the angle of rotation of $\mathbb{R}$ is $60^\circ$ . [1]

(ii) Name the image of point B under $\mathbb{R}$ . [1]

(iii) Name the image of the *line* BZ under $\mathbb{R}$ . [1]

(b) If H is the image of point G under $\mathbb{R}$ , explain why

(i) triangle AGH is equilateral, [2]

(ii) H lies on the line CY. [1]

(c) Explain why

(i) AG=GH, [1]

(ii) BG=YH. [1]

(d) Name two lines, *each* of which is equal in length to AG+BG+CG. [2]



**[End of Paper]**