# SIMULTANEOUS EQUATIONS

## REVISE

Simultaneous equations are a pair of equations where there is more than one variable (unknown values).

Let's look at the method to solve these types of equations with the help of a few examples.

**Example:** Solve the simultaneous equations.

$$3x + y = 9$$

$$2x - y = 1$$

**Solution:**

To solve the equations, number them as 1 and 2.

$$3x + y = 9 \ldots (1)$$

$$2x - y = 1 \ldots (2)$$

Check the numbers in front of $x$ or $y$.

Here the numbers in front of $x$ are 2 and 3 and in front of $y$ are 1 and 1.

> Ignore the signs of the numbers for this step

Add / Subtract the two equations to remove one of the terms.

In this case, we must add equations (1) and (2) to remove the $y$ term.

$$\begin{array}{r} 3x + y = 9 \\ +\ \ 2x - y = 1 \\ \hline 5x = 10 \end{array}$$

> If we had same signs for y, we would do subtraction

Now solve the equation with one unknown using the **balance method**.

$5x = 10$

$5x \div 5 = 10 \div 5$

$x = 2$

Last step is to substitute the answer back into one of the equations, say (1) in this case.

$3 \times 2 + y = 9$

$6 + y = 9$

$6 - 6 + y = 9 - 6$

$y = 3$

**Example:** Solve the simultaneous equations.

$$2x + 3y = 13$$
$$3x + y = 9$$

**Solution:**

Let us first number the equations as 1 and 2.

$2x + 3y = 13 \ldots (1)$

$3x + y = 9 \ldots (2)$

Here the numbers in front of $x$ are 2 and 3 and in front of $y$ are 3 and 1.

So neither $x$ nor $y$ can be removed by adding or subtracting.

To remove one of them, multiply equation (2) by 3.

$3 \times (3x + y = 9)$ which gives us $9x + 3y = 27 \ldots (3)$

Renumber the equation as (3).

Now Subtract equation (1) from (3).

When you multiply the equation make sure you multiply each of the terms

$$\begin{array}{r} 9x + 3y = 27 \\ -\ 2x + 3y = 13 \\ \hline 7x = 14 \end{array}$$

So we get $x = 2$,

Substitute $x = 2$ in equation (2), $3 \times 2 + y = 9$

$6 + y = 9$ which gives $y = 3$

**Example:** 3 bananas and 4 lemons cost 78p. 4 bananas and 3 lemons cost 76p. What is the cost of a lemon?

**Solution:**

Assume the cost of one banana as $x$ and the cost of one lemon as $y$.

Recall the three steps to solve algebraic problems

Cost of 3 bananas and 4 lemons = $3x + 4y = 78$p… (1)

Cost of 4 bananas and 3 lemons = $4x + 3y = 76$p… (2)

Multiplying (1) by 3, gives
$9x + 12y = 234$… (3)

We are multiplying to make the number in front of y the same

Multiplying (2) by 4, gives
$16x + 12y = 304$… (4)

Subtracting equation (3) from (4),

$$\begin{array}{r} 16x + 12y = 304 \\ -\quad 9x + 12y = 234 \\ \hline 7x = 70 \end{array}$$

So we get, $x = 10$

Substitute $x = 10$ in equation (1),

$3 \times 10 + 4y = 78$

$30 + 4y = 78$

$4y = 48$

$y = 12$

The cost of one banana is 10p and the cost of one lemon is 12p.

## PRACTICE

1 **A chocolate ice cream and a vanilla smoothie together cost £1.25. If I buy a chocolate ice cream and 5 vanilla smoothies I pay £3.25. Work out the cost of chocolate ice cream and vanilla smoothie respectively.**

A. 75p and 40p

B. 85p and 40p

C. 75p and 50p

D. 65p and 55p

E. 65p and 60p

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Assume the cost of chocolate ice cream to be £$x$

Assume the cost of vanilla smoothie to be £$y$

$x + y = 1.25$ ... (1)

$x + 5y = 3.25$ ... (2)

Subtracting equation (1) from (2)

$$\begin{array}{r} x + 5y = 3.25 \\ -\quad x + y = 1.25 \\ \hline 4y = 2.00 \end{array}$$

$y = 2 \div 4$

$y = 0.5$

Using the value of $y = 0.5$ in (1) we get,

$x + 0.5 = 1.25$

$x = 1.25 - 0.5$

$x = 0.75$

The cost of one chocolate ice cream = £0.75 = 75p

The cost of one vanilla smoothie = £0.5 = 50p

The correct answer is Option C

**2** **In a Tribonacci sequence, each number after the third one is the sum of the previous three numbers. For example, if the first three numbers are 0, 1, and 2 then the sequence is 0, 1, 2, 3, 6, 11, .... Find the 4th term of the Tribonacci sequence given below.**

1, _____ , 10, _____ , 29

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Assume the $2^{nd}$ term as $x$ and the $4^{th}$ term as $y$

Using the condition,

next number $=$ sum of previous three numbers

$y = 1 + x + 10$

$y - x = 11 \quad ...(1)$

$29 = y + 10 + x$

$y + x = 19 \quad ...(2)$

Adding equations (1) and (2)

$$\begin{array}{rr} & y - x = 11 \\ + & \underline{y + x = 19} \\ & 2y = 30 \\ & y = 30 \div 2 \\ & y = 15 \end{array}$$

Hence the $4^{th}$ term $= \underline{15}$

**3** **Dad's wallet contains only 10 coins consisting of 5p and 10p coins. The total value in his wallet is £0.75. How many coins of each type are there in his wallet?**

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Assume number of 5p coins as $x$ and number of 10p coins as $y$

$x + y = 10 \quad ...(1)$

Total value of coins $=$ £0.75 $=$ 75p

Value of 5p coins $= 5 \times x = 5x$

Value of 10p coins $= 10 \times y = 10y$

$5x + 10y = 75 \quad ...(2)$

Multiplying (1) by 5, $5 \times (x + y = 10)$

$$5x + 5y = 50 \quad ...(3)$$

Subtracting (3) from (2),

$$\begin{array}{rr} & 5x + 10y = 75 \\ - & 5x + 5y = 50 \\ \hline & 5y = 25 \end{array}$$

$$y = 25 \div 5$$

$$y = 5$$

Using the value of $y = 5$ in (1) we get,

$x + 5 = 10$

$x = 10 - 5$

$x = 5$

Number of 5p coins $=$ 5 and number of 10p coins $=$ 5

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| **Exam Tips** | • After you find the unknowns make sure you substitute the values in both the equations to check your answer. |
| **Keypoints** | • To solve simultaneous equations match the numbers in front of two unknowns and add/subtract the equations to remove one of the unknowns.<br>• To solve algebraic word problems, always mark the important information required to make an equation. |